Gandhi Memorial College Of Education Bantalab Jammu

INFORMATICS

MATHEMATICS FOR COMPUTING (RM104)

Study Guide

Gandhi Memorial College Of Education Bantalab Jammu

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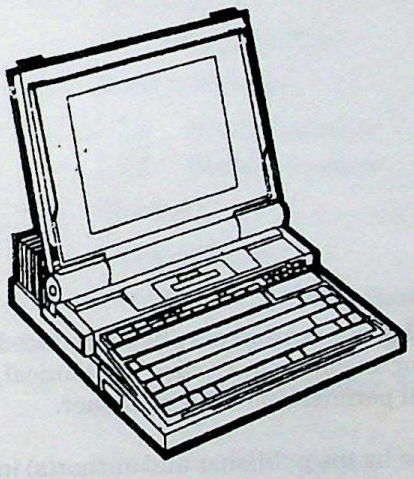

MATHEMATICS FOR COMPUTING (RM104)

Study Guide

4th Edition

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Chapter 1

Number Bases

Chapter Objectives

At the completion of this chapter, you should be able to:

- identify the different types of numbers;
- convert between denary and other base;
- convert between binary, octal and hexadecimal;
- understand the column system and number bases;
- understand modular arithmetic;
- hexadecimal, octal addition.

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1.1 Introduction

In this chapter, we start to explore various sets of numbers which are used to represent data in real life problems. Namely, natural numbers, integers, rational, irrational, real and complex numbers. Numbers are used to represent quantities, measurement, and others in our surroundings. They are essential representations of data that will be processed by computer systems to produce useful information. The computer system, being a digital electronic device, has to handle data in binary numbers.

In binary numbers, each binary digit has only two states rather than ten states in decimal numbers. Therefore, binary, octal and hexadecimal number systems are dealt with in subsequent parts of the chapter.

1.1.1 Number Sets

N	Natural Positive integers Example: 3, 71
Z	Integers Whole numbers Example: 3, -71
Q	Rational Real numbers which can be expressed as the ratio of 2 integer Example: 1/2, 0.57, -3
F	Irrational Real numbers which are not rational Example: $\sqrt{2}$, π
R	Real Can be represented by points a the straight line Example: -2.31, $\sqrt{5}$, 6
C	Complex No real number that can satisfy the equation $X^2 = -1$ Example: $\sqrt{-1}$, $\sqrt{-5.75}$, $\sqrt{-88}$

1.2 Level of Precisions

Some translator software have two levels of precisions in storing real numbers. Namely single-precision and double precision.

1.2.1 Single Precision

The real number is calculated to 8 and displayed to 7 significant figures.

Example: π = 3.1415926 calculated
= 3.141593 displayed

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1.2.2 Double Precision

The computer will need longer processing time to produce double precision numbers. The real number is calculated to 16 and displayed on the screen to 15 significant figures.

Example: π = 3.141592653589793 calculated
= 3.14159265358979 displayed

1.3 Number Base

A fundamental counting group consists of a number of various symbols.

Counting Groups	Symbols of the Group	No. of Symbols
Binary	0, 1	2
Octal	0, 1, 2, 3, 4, 5, 6, 7	8
Hexadecimal	0, 1, 2, 3, ... 9, A, B, C, D, E, F	16
Denary	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	10

1.4 Column System

Enable us to represent any conceivable number by combining numerals and zeros.

For example, 423524 is derived from this column system;

Column No	6	5	4	3	2	1
Value	100,000	10,000	1,000	100	10	1
Power of Magnification	10^5	10^4	10^3	10^2	10^1	10^0
Numerals	4	2	3	5	2	4

$423524 = 4 \times 10^5 + 2 \times 10^4 + 3 \times 10^3 + 5 \times 10^2 + 2 \times 10^1 + 4 \times 10^0$

The Power of Magnification for each individual column is derived as,
Power of Magnification = $\text{Base}^{(\text{Column no} - 1)}$

1.5 Conversion from Other Bases to Denary

We can use this column system to convert a number in other bases to its equivalence in Denary.

Example: Convert 111001_2 to denary

Column No	6	5	4	3	2	1
Value	32	16	8	4	2	1
Power of Magnification	2^5	2^4	2^3	2^2	2^1	2^0
Numerals	1	1	1	0	0	1

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$111001_2 = 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^0$
$= 32 + 16 + 8 + 1$
$= 57_{10}$

Example: Convert $4BEEF8_{16}$ to denary

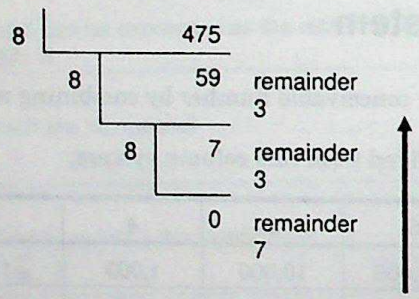
Column No	6	5	4	3	2	1
Value	1048576	65536	4096	256	16	1
Power of Magnification	16^5	16^4	16^3	16^2	16^1	16^0
Numerals	4	B	E	E	F	8

$4BEEF8_{16} = 4 \times 16^5 + 11 \times 16^4 + 14 \times 16^3 + 14 \times 16^2 + 15 \times 16^1 + 8 \times 16^0$
$= 4976496_{10}$

1.6 Conversion from Denary to Other Bases

To convert a denary number to other bases, we do a repeated division by the desired base until a quotient 0 is obtained.

Example: Convert 475_{10} to octal



The octal number is obtained by reading the last integer 7 and upwards to include all reminders; $475_{10} = 733_8$

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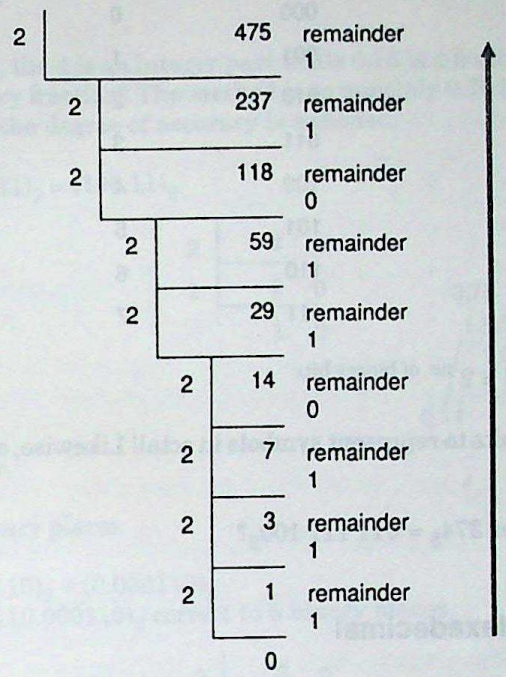
Example: Convert 475_{10} to binary



$$475_{10} = 111011011_2$$

1.7 Conversion Among Other Base

We have illustrated the conversions between denary and other bases. How about converting a binary number to an octal number? Or, an octal number to a hexadecimal one?

1.7.1 Binary to Octal

We know that $9_{10} = 1001_2$ and $9_{10} = 11_8$ so, we can conclude that $1001_2 = 11_8$. To convert a binary number to an octal number in this case will involve a lot of calculations.

Is there a better way to convert these numbers?

Yes. How many binary bits do we need to represent an octal digit?

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Binary	Octal
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

No. of symbols in octal = $2^{\text{no. of binary bits}}$

We need three binary bits to represent symbols in octal! Likewise, one octal number will produce three binary bits.

To prove it, try whether $374_8 = 011\ 111\ 100_2$?

1.7.2 Binary to Hexadecimal

Following the same argument, it is not so difficult to see why we need to have four binary bits to represent one hexadecimal number.

No. of symbols in Hexadecimal = $2^{\text{no. of binary bits}}$

Now can you see why $110101100_2 = 1AC_{16}$?

0001	1010	1100
1	A	C

1.7.3 Conversion Between Octal and Hexadecimal

Since every octal number will produce three binary bits, and every four binary bits will produce one hexadecimal number. We can make use of the binary base as the "conversion medium" to convert a hexadecimal number to an octal number or vice versa.

Example: Convert 733_8 to a hex no.

Step 1. Convert 733_8 to a binary number.
111 011 011

Step 2. Rearrange this binary number into 4-bit groups.
0001 1101 1011

Step 3. Convert this binary number to a hex.
1 D B
Hence, $733_8 = 1DB_{16}$

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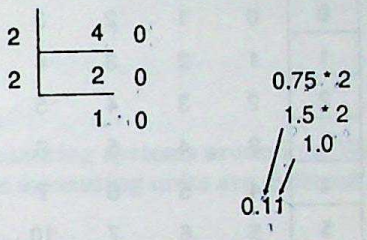
1.8 Real Number

In real numbers, e.g. $(4.75)_{10}$, the 4 is an integer part while 0.75 is a fractional part. What is the way to convert 0.75 to a binary fraction? The method is to multiply 0.75 by 2 continuously until the fraction becomes zero or the degree of accuracy is satisfied.

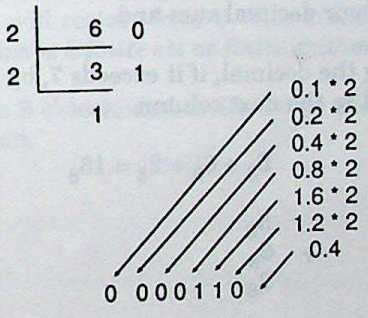
$4.75 = 4 + 0.75 = (100)_2 + (0.11)_2 = (100.11)_2$



Let's look at another example.

$(6.1)_{10}$ to be converted to binary places.

6.1 = 6 + 0.1 = $(110)_2 + (0.000110)_2$
= $(110.000110)_2$ correct to 6 binary places.



On the other hand, how can we convert $(110.000110)_2$ to denary real numbers?

Method

i. Write down the value of each bit (binary digit)

ii. Multiply the value by every bit

iii. Take the sum of products

4	2	1	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
1	1	0	.0	0	0	1	1	0

4*1+2*1+0*1+0*0.5+0*0.25+0*0.125+1*0.0625+1*0.03125+0*0.015625

= 6.09375

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1.9 Octal Arithmetic

We only cover octal addition. The sum of two octal numbers can be reduced by the usual addition algorithm to the repeated addition of two digits (with possibly a carry of 1). The following table show the addition of Octal number.

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

The sum of two octal digits, or the sum of two octal digits plus 1, can be obtained by :

i. Finding their decimal sum and

ii. modifying the decimal, if it exceeds 7, by subtracting 8 and carrying 1 to the next column.

Example : $5_8 + 6_8 + 2_8 = 15_8$

$$\begin{array}{rr} & 5_8 \\ + & 6_8 \\ & 2_8 \end{array}$$

Decimal sum	13
Modification -	8
Octal sum	15_8

1.10 Hexadecimal Arithmetic

As with the octal system, we cover only hexadecimal addition.

The sum of two hexadecimal digits, or the sum of two hexadecimal digits plus 1, can be obtained by:

i. Finding their decimal sum and

ii. modifying the decimal, if it exceeds 15, by subtracting 16 and carrying 1 to the next column.

If the base exceeds ten, we need mentally to change each hexadecimal letter digit to its decimal form when finding the decimal sum, and each decimal difference greater than nine to its hexadecimal form when modifying the decimal sum.

A = 10 B = 11 C = 12 D = 13 E = 14 F = 15

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Example : $A_{16} + 9_{16}$

$$\begin{array}{r} A_{16} \\ + \ 9_{16} \\ \hline \end{array}$$

Decimal sum	19
Modification -	16
Octal sum	13_{16}

1.11 Modular Arithmetic

In our daily life, there are many counting/measuring systems around us. We know that 100cm is not the same as 100 inches. It's because the measuring units are different, however, we don't intend to cover the conversions of this kind.

To demonstrate how *modular arithmetic* works is to give a test first:

"If Peter starts work at 8 o'clock in the morning and work for 8 hours, at what time will Peter finish work?"

4 o'clock in the afternoon, right? But how you worked that one out? Because the clock only has 12 hours, once the short-hand reaches 12, it will restart from 0. The numbers we see on the clock-face must be less than or equal to 12. This is a *finite set* or *finite arithmetics.*

To show it mathematically, we add 8 hours to 8 o'clock, and divide 16 by the *modules* number 12, the remainder 4 will be the answer we want.

$$8 + 8 = 16$$
$$(16)_{\text{mod } 12} = 4$$

Example: $(11 + 3 + 7 + 9)_{\text{mod } 12}$

$$30/12 = 2 \text{ remainder } 6$$
$$30 \text{ mod}_{12} = 6$$

Points to Remember

- ❒ The various sets of numbers include:
  - Natural numbers
  - Integers
  - Rational numbers
  - Irrational numbers
  - Real numbers
  - Complex numbers
- ❒ The higher the precision required the longer the processing time in computer systems.

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- 4 number systems
  - Decimal (Denary)
  - Binary
  - Octal
  - Hexadecimal
- Additional of Octal and Hexadecimal
- Convert from other base to *decimal* integer

  Step 1. Write down the weight of each digit;

  Step 2. Multiply each weight and each digit;

  Step 3. Take the sum of the product.
- Convert from *decimal* integer to other bases

  Step 1. Divide the decimal integer by the desired base;

  Step 2. Write down the remainder;

  Step 3. Repeat dividing until a quotient 0;

  Step 4. Read the remainders from bottom upwards.
- *Octal* and *hexadecimal* numbers are used as a shorthand for *binary* numbers.
- Each *octal* digit can be expressed as 3 *binary* digits.

  Each *hexadecimal* digit can be expressed as 4 *binary* digits.
- Use *modular* arithmetic when the data is finite.

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Past Year Questions

1. Express the number 747_8 in :
   a. Binary [1]
   b. Denary [1]
   c. Hexadecimal [1]

2. Convert showing all working
   a. 21.625 denary to binary [1]
   b. 2AE hexadecimal to denary [1]
   c. 16.62 octal to binary [1]
   d. 567 octal to binary [1]
   e. 684 denary to hexadecimal [1]

3. Convert the following :
   a. 157 denary to binary [1]
   b. 1100110101 binary to octal [1]
   c. ACD hexadecimal to denary [1]
   d. 2464 octal to hexadecimal [1]

4. Convert the following :
   a. 101 101 101 Binary to Octal [1]
   b. DAB Hexadecimal to Denary [1]
   c. 2839 Denary to Hexadecimal [1]
   d. 7453 Octal to Hexadecimal [1]

5. Express the denary number 567
   a. in binary [1]
   b. in hexadecimal [1]

6. Express the number 103_8
   a. in binary [1]
   b. hexadecimal [1]

7. Convert :
   a. 274 Octal to DENARY [1]
   b. DA3 Hexadecimal to OCTAL [1]

8. Convert :
   a. ABC Hexadecimal to OCTAL [1]
   b. 3974 Denary to HEXADECIMAL [1]

9. Convert :
   a. 7456 Octal to HEXADECIMAL [1]
   b. 9E7 Hexadecimal to DENARY [1]

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10. Convert :
    a. 8543 Denary to OCTAL [1]
    b. 9AD Hexadecimal to OCTAL [1]

11. Convert :
    a. A25 HEXADECIMAL to BINARY [1]
    b. 549 DENARY to OCTAL [1]
    c. 3527 OCTAL to HEXADECIMAL [1]

12. Convert :
    a. 5391 Denary to HEXADECIMAL [1]
    b. 6A5 Hexadecimal to OCTAL [1]

13. Convert :
    a. 5743 Denary to HEXADECIMAL [1]
    b. ABC Hexadecimal to OCTAL [1]

14. Convert the following : (You MUST show all workings)
    a. 11010010_2 to Hexadecimal [2]
    b. $54A_{16}$ to Denary [2]
    c. 1017_8 to Binary [2]
    d. 167_{10} to Binary [2]

15. Convert the following:
    a. 175264_8 to base 16 [1]
    b. 1101_{10} to base 16 [1]
    c. $72_8 + 2_8$ to base 10 [2]
    d. $B00_{16} + 1F_{16}$ to base 2 [2]

16. a. Express the number 23_{10} in binary [1]
    b. Express the number 46_{10} in binary [1]
    c. Express the number 92_{10} in binary [1]
    d. Express in binary the result of multiplying 100110111_2 by 4_{10} [2]

17. Convert the following :
    a. 357_8 to base 16 [2]
    b. 1011_{10} to base 16 [2]
    c. $504_8 + 2_8$ to base 10 [2]
    d. $6A_{16} + D00_{16}$ to base 2 [2]

18. What is the BASE of the number system where 36 + 27 = 65 ? [2]

19. By converting to BINARY, evaluate the
    HEXADECIMAL expression : 7B + EA.
    Give your answer in HEXADECIMAL. [2]

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20. a. Evaluate $(7 * 4 + 6 * 5)_{mod\ 11}$ [1]

b. Solve $(3 * p = 8)_{mod\ 11}$ [2]

21. a. Evaluate $(7 * 3 + 5 * 2 + 2 * 1)_{mod\ 11}$ [1]

b. Evaluate $(3 * p)_{mod\ 5}$ for p = 0, 2 and 4 [2]

22. a. Evaluate $(8 * 5 + 7 * 6)_{mod\ 11}$ [1]

b. Solve $(3p = 7)_{mod\ 11}$ [2]

23. Evaluate ((4*6) + (35 DIV 4)) MOD 11 [3]

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Chapter 2

Computer Based Arithmetic

Chapter Objectives

At the completion of this chapter, you should understand how to:

- perform binary calculation;
- store numbers into computer words;
  - integer
  - fraction
  - mixed number;
- store numbers into computer words using;
  - sign modulus method
  - 2's complement method;
- perform 2's complement subtraction;
- shift operations.

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2.1 Binary Addition

Binary addition for fixed point numbers is done as follows:

- The numbers to be added are aligned by their binary points.
- Starting from the least significant (rightmost) digit, a corresponding pair of digits (in the same column) are added according to the rules of binary addition.
- If a column has a carry from the previous column, the carry digit has to be added also.
- Repeat steps (ii) and (iii) for each column until the most significant (leftmost) digit have been added.

2.1.1 Rules for Binary Addition

1+1	=	0	← and carry 1 to add to next column (i.e. 1+1 = 10)
1+0	=	1	
0+1	=	1	
0+0	=	0	

2.1.2 Examples of Binary Addition

Example:

	Binary		Decimal equivalent
	1010		10
+	0111	+	7
	10001		17

Example:

	Binary		Decimal equivalent
	1010.01		10.25
+	0110.11	+	6.75
	10001.00		17.00

Example:

	Binary		Decimal equivalent
	111 ← carry		1 ← carry
	101100		44
+	011101	+	29
	1001001		73

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Example:

Binary		Decimal equivalent	
11 ← carry			
	0.11		0.75
+	0.01	+	0.25
	1.00		1.00

2.2 Binary Subtraction

2.2.1 Rules for Binary Subtraction

0-0	=	0	
1-1	=	0	
1-0	=	1	
0-1	=	1	← with borrow "1" from the column to the left of the number subtracted from

2.2.2 Examples of Binary Subtraction

Example:

Binary		Decimal equivalent	
	11010		26
-	01001	-	9
	10001		17

Example:

Binary		Decimal equivalent	
	1010.00		10.00
-	1000.11	-	8.75
	00001.01		1.25

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The subtraction process can be more clearly illustrated in the following example.

Example:

	Step 1	Step 2	Step 3	Step 4
	1	11	011	011
11000	11000	11000	11000	11000
- 00001	- 00001	- 00001	- 00001	- 00001
	1	1	1	10111
	Borrow changes 0 to 1	Continue changing 0's to 1's	Until a 1 can be changed to a 0, then proceed with subtraction	

2.2.3 Examples on Successive Borrows in Binary Subtractions

Example: $8_{10}-2_{10}=6_{10}$

0	1				
1	0	0	0	=	8_{10}
0	0	1	0	=	2_{10}
0	1	1	0	=	6_{10}

Example: $9_{10}-7_{10} = 2_{10}$

0	1				
1	0	0	1	=	9_{10}
0	1	1	1	=	7_{10}
0	0	1	0	=	2_{10}

2.3 Binary Multiplication

	A	=	Multiplicand
x	B	=	Multiplier
	C	=	Product

2.3.1 Rules for Binary Multiplication

0x0	=	0
0x1	=	0
1x0	=	0
1x1	=	1

Copy the multiplicand when the multiplier digit is 1; do not when it is 0. Shift as in decimal multiplication. Add the resulting binary numbers according to the binary addition rules.

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2.3.2 Examples of Binary Multiplication

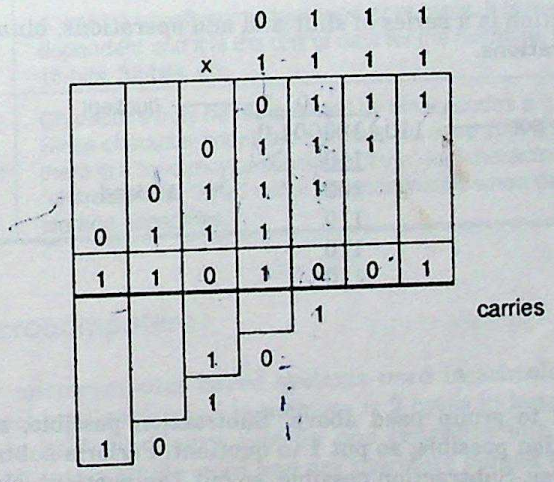
Example:

Binary	Decimal equivalent
1101	13
x 1100	x 12
0000	26
0000	13
1101	156
1101	
10011100	

Example:

Binary	Decimal equivalent
101	5
x 111	x 7
101	35
101	
101	
100011	

2.3.3 Handling of Carries in Multiplication



The product is 1101001_2

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Example: $4_{10} \times 3_{10} = 12_{10}$

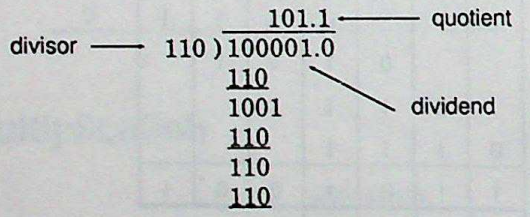
$$
\begin{array}{rrrrrrrrl}
 & & & 0 & 1 & 0 & 0_2 & = & 4_{10} \\
\times & & & 0 & 0 & 1 & 1_2 & = & 3_{10} \\
\hline
 & & & 0 & 1 & 0 & 0 & & \\
 & & 0 & 1 & 0 & 0 & & & \\
 & 0 & 0 & 0 & 0 & & & & \\
0 & 0 & 0 & 0 & & & & & \\
\hline
0 & 0 & 0 & 1 & 1 & 0 & 0_2 & = & 12_{10}
\end{array}
$$

Example: $10_{10} \times 14_{10} = 140_{10}$

$$
\begin{array}{rrrrrrrrrl}
 & & & & 1 & 0 & 1 & 0_2 & = & 1010 \\
 & & x & & 1 & 1 & 1 & 0_2 & = & 14_{10} \\
\hline
 & & & & 0 & 0 & 0 & 0 & & \\
 & & & 1 & 0 & 1 & 0 & & & \\
 & & 1 & 0 & 1 & 0 & & & & \\
 & 1 & 0 & 1 & 0 & & & & & \\
\hline
1 & 0 & 0 & 0 & 1 & 1 & 0 & 0_2 & = & 140_{10}
\end{array}
$$

2.4 Binary Division

Just as binary multiplication is a series of shift and add operations, binary division is a series of shift and subtract operations.



Binary

Add digit from dividend to group used above. Subtraction possible, so put 1 in quotient. Remainder from subtraction possible, so put 1 in quotient. Perform subtraction, and add digit from dividend to remainder. Subtraction possible, so put 1 in quotient. No remainder, so stop.

Decimal Equivalent

$$
\begin{array}{r}
5.5 \\
\hline
6 \,)\, 33 \\
30 \\
\hline
30 \\
30 \\
\hline
\end{array}
$$

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2.4.1 Example of Binary Division

Example: $\frac{32_{10}}{3_{10}} = (10^2/_3)_{10}$ or 10 remainder 2

```
          1010.10
11 ) 100000.00
       11
        100
         11
         100
          11
          10   remainder
```

2.5 Number Storage in the Computer Word

Computer storage is arranged in words. A word is defined as a unit of data for the machine. Words vary in length on different types of machines. Common word sizes are 8, 12, 16, 24 and 32 bits.

Bit	A 'bit', short for binary digit, is in fact the smallest unit of information stored in a circuit element such as a 'gate' or moved through the circuitry as a 'pulse' or 'no-pulse'. By itself, it is of course limited in its capacity to carry information.
Byte	A 8-bit string of binary bits is known as a byte. It is widely used because it corresponds to one character.
Word	A word is usually a multiple number of bytes. It is machine dependent and it is the unit of data for the machine. e.g. 8-bits, 16-bits, 32-bits, etc.
Character	Characters may be represented by binary codes e.g. ASCII code. Since character codes are normally no more than 8 bits in length, there is a tendency to substitute "byte" for "character" and "character" for "byte" rather indiscriminately when describing storage capacities.

2.5.1 16 Bit Microcomputers

Most of the popular microcomputer based systems used in schools and colleges are "16-bit micros". In these "machines" each word in memory is 2 bytes in length (i.e. 8-bits). Each word is sufficiently large to hold the binary code for one character but program instructions are normally too long to fit into a single word and are therefore spread over two or three words in sequence.

2.5.2 32 Bit Machines

Some of the more modern microcomputers are "32-bit micros". Many mini-computers are also "32-bit machines". Each word is four bytes (32-bits) in length, i.e. the "word length" is 32-bits.

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2.5.3 Size Limits of Data

Computer storage is of finite size. This limits the size of numbers which can be stored and their accuracy.

Example: 16 bit words are common and hold numbers from -32768 to 32767.

2.6 Storage of Numbers

It is important to know the format of storage before interpreting the given 0's and 1's stored in a computer word. There are many formats of storing numbers:

- ❒ Integer (using sign modulus format)
- ❒ Integer (using 2's complement format)
- ❒ Fraction

} Fixed point format

- ❒ Mixed number
- ❒ Floating point number

2.6.1 Storage of Integers Using Sign Modulas Method

- ❒ Types of numbers
  - Even if you know that the data is numeric, make sure you have the right format

① ② ③

→ ← → ←

[00110101 | 11000000]

③	=	13760	decimal integer
①	=	0.2099609375	decimal fraction
②	=	53.75	decimal mixed

and there are many other formats

2.6.2 Storage of Integers

- ❒ 735 decimal = 1011011111 (10 bits)
- ❒ Add 2 bits "padding" in a 12 bit word

0	0	1	0	1	1	0	1	1	1	1	1

- ❒ By convention, the first (left-hand) bit is the *sign bit*
- ❒ Therefor only 11 bits are left for the number value

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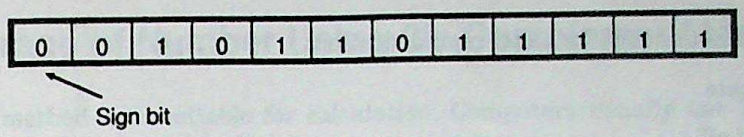
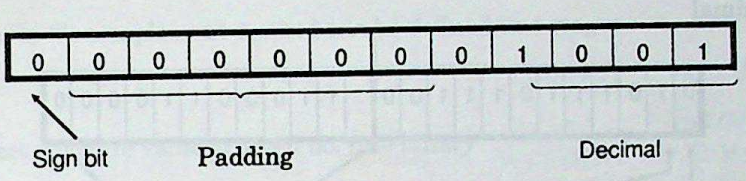
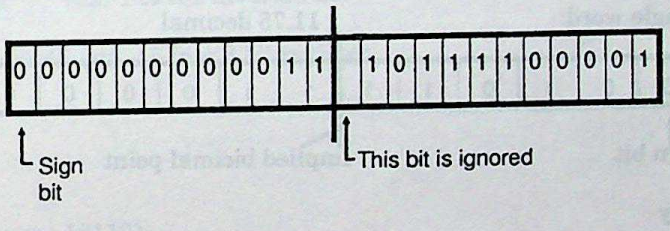


For sign modulus method 0 ⟶ Positive
1 ⟶ Negative

- Small numbers stored in 12-bit words are mostly "padding" bits



- Padding is not really wasteful, it is necessary to make the calculation methods work properly
- The upper limit for storing a number in a 12-bit word is 011111111111 = 2047 decimal
- The range of a given computer n bit word for sign modulus is $-(2^{n-1})$ to $(2^{n-1}-1)$
- Larger numbers need double length



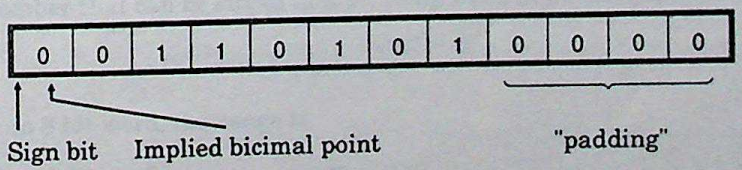
7649 decimal = 1110111100001

22 bits can hold 4194303_{10}

2.7 Storage of Fractions

- First bit still sign bit
- Bicimal point not stored
  - Implied after sign bit

Decimal 0.4140625



This representation is exact.

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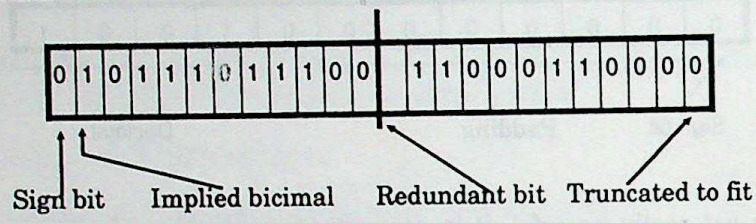
- If 11 bits is not enough for exact representation:
  - Truncate
  - Round off
  - Extend to double length (or more)
- Double length fraction:

  0.7328 decimal

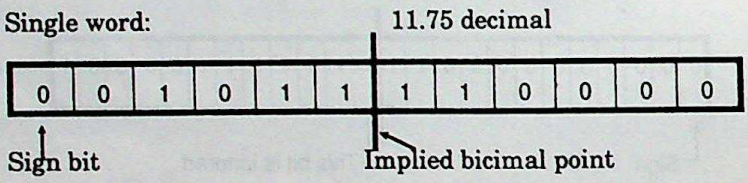


  - Double length fractions have increased accuracy, not range

2.8 Storage of Mixed Numbers

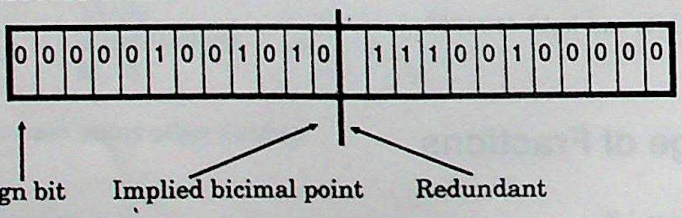
(Fixed Point Notation)

Single word: 11.75 decimal



Divide as necessary

Double word: 74.890625 decimal



- Usual convention is one word for integral part, the other for fraction

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2.9 Storage of Number Using 2's Complement Method

Sign-modulus method is unsuitable for calculation. Computers usually use 2's complement method to store numbers and calculation.

In 2's complement method, no sign bit is reserved to indicate the sign of number. However, if the leftmost bit is 0, it indicates a positive number. If the leftmost is a 1, then it is an implied negative number.

Stored numbers in 2's complement method can be defined in 4 stages.

II Method:
Start traversing from R.H.S,
101011010
=>010100110

Steps:

1. Convert absolute value of given no. into binary
2. Store binary no. into given computer word
3. Check if given no is positive, the steps are completed
4. If given no is negative then

   4.1 Invert all bits
   $0 \rightarrow 1$
   $1 \rightarrow 0$

   4.2 Add 1 to the inverted no.

Example: Store 45 and -45 into 8 bit register

To store 45:

Steps:

1. 45 in binary : 101101
2. 45 in 8 bit : 00101101
3. Since no. (45) is positive, task is completed.

To store -45:

Steps:

1. 45 in binary : 101101
2. 45 in 8 bits: 00101101
3. -45: 11010010
   $+ \quad 1$
   11010011

The range of number that can be stored in n-bit comp word in 2's complement format is

$$-(2^{n-1}) \text{ to } (2^{n-1}-1)$$

Example: in 8 bit word, the range is

$-(2^{8-1})$ to $(2^{8-1}-1)$ = -128 to 127

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2.10 Ten's Complement

8' = 2 because 8 + 2 = 10 as such

8 - 2 = (8 + 8) - 10 = 6
6 - 4 = (6 + 6) - 10 = 2

This findings has a significant impact on the arithmatics operation in computer.

2.11 Two's Complement

Computers can only do addition, so subtraction is done by adding the complement of the subtrahend to the minuend.

(Using four bits to store the binary number)

Example:

3	convert to binary	0101		
-5	convert to binary -	0011		
-2		1100	→	step 1 invert the bits of 0011
		+ 1	→	step 2 add 1 to the least significant bit of 1100
		1101	→	step 3 add the 2's complement of 0011 to 0101
	+	0101		
	carry 1 ignored	0010	→	convert to denary is 2

2.12 Recomplementing Cases

If the minuend is lesser than the subtrahend then it is necessary to recomplement the answer and add a minus sign to it.

Example:

3	convert to binary	0011		
-5	convert to binary	0101		
-2		1010	→	step 1 invert the bit of 0101
		+ 1	→	step 2 add 1 to the least significant bit of 1010
		1011	→	step 3 add the 2's complement of 0101 to 0011
		0011		
		1110	→	step 4 recomplement 1110 and add a minus sign
		0001		to the answer
		+ 1		
		-0010	→	convert to denary is -2

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2.13 Shift Operations

Shifting refers to a process whereby it is possible to physically move the pattern of bits within a computer word to the left or to the right. A number of different types of shift operation do exist. The most common are circular, logical and arithmetic shift. We will only be interested in the arithmetic shift operation.

2.13.1 Shift Operations to Achieve Multiplication

If every bit in a 12-bit word is moved one place to the left it immediately has an associated place-value which is twice that previously associated with it and so the effect of shifting one place to the left is to double the value of the quantity originally stored. If shifted two places to the left, this doubling is repeated, so that the result is now four times the original; three place shifted to the left has the effect of multiplying the quantity by eight (since $8 = 2^3$) and so on.

Under these circumstances the gaps which occur at the right hand end of the word as the shifting takes place are filled with zeros and anything shifted beyond the left-hand end is lost. Additionally, this process is only valid so long as the sign-bit remains as zero, (although techniques do exist to cope with negative quantities). But if a positive number is to be multiplied in this way and there is possible to avoid the difficulty by going into double-length.

Example:

original binary number

0	0	0	0	0	0	1	1	0	0	1	0

+ 50

← 1 place shift to the left

0	0	0	0	0	1	1	0	0	1	0	0

+ 100 = (50×2^1)

← ← 2 places shift to the left

0	0	0	0	1	1	0	0	1	0	0	0

+ 200 = (50×2^2)

← ← ← 3 places shift to the left

0	0	0	1	1	0	0	1	0	0	0	0

+ 400 = (50×2^3)

2.13.2 Shift Operations to Achieve Division

Just as shifting to the left effects multiplication by the relevant power of two, so shifting to the right achieves division by the relevant power of two. In this case the gaps caused by shifting occur at the left-hand end of the word and are replaced by copies of the sign-bit to retain arithmetic significance: it is in fact more straightforward to cope with negative values here than in the case of multiplication. Any bit shifted beyond the right-hand end of the word is lost and the result is a truncated form of division.

If the last bit to be shifted beyond the right-hand end of the word were to be then added back on to the right-hand end bit the result would be a rounded form of the same division.

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Past Years Questions

1. If a computer stores integers in one byte using the two's complement:
   a. What range of integers can be represented? [1]
   b. Show how the computer would evaluate -100 -40.
   You must show all working. [5]
   c. Briefly comment on your results in (b). [1]

2 a. Determine the range of integer that could be stored in a 6 bits register in two's complement form.
   b. Using two's complement, evaluate the following numbers which are stored in 5 bits registers, evaluate the following:
      i. 12 - 15 [2]
      ii. 15 - 10 [2]

3. A particular computer's 8 bit registers store number in two's complement form.
   a. Show how -96 is represented. [1]
   b. Show how 85 -120 would be evaluated. [4]

4. a. Express 101.101 and -100.1$\dot{1}$ in binary using 8 bits for the integer part and 8 bits for the (unrounded) fraction part. [4]
   b. Convert the fraction 10011100.10100110 to decimal. [4]

5. Calculate the following using complement subtraction, assuming the numbers are stored in 8-bit registers 13 - 56. [4]

6. a. If numbers are held in an 8 BIT register in TWO's COMPLEMENT form.
      i. Write down the COMPLETE range of INTEGERS which can be held. [2]
      ii. Show how 96 - 124 would be evaluated. [4]
   b. If a very simple 4 BIT register were to be used.
      i. What would the binary representation of 0.7_{10} be? [2]
      ii. What denary value would the register actually hold? [1]

7. a. Show, in BINARY, how a 12-BIT register would hold.
      i. The integer -894 using TWO's complement [4]
      ii. Show how -894 would be added to ITSELF using TWO's complement and justify that the CORRECT answer is obtained [3]
      iii. Show what happens if -894 is added yet again. [1]
      iv. What is this situation called? [1]

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8. Using TWO's COMPLEMENT, base 10 integers are to be stored in an EIGHT bit register

   a. What range of INTEGERS, (expressed in DENARY form) can be stored? [2]

   b. Show how 71_{10} is represented [1]

   c. Show how -83_{10} is represented [2]

   d. Show how 71_{10} -83_{10} would be evaluated [3]

9. a. If a computer stores integers in two bytes using TWO's complement

      i. What is the range of integers which can be represented? [2]

      ii. How would the computer evaluate 47 -125? Show all your working. [4]

   b. Given 1001 0011 0101 in a 12-bit word, interpret this in denary if it is of format:

      i. TWO's complement [3]

      ii. Fixed point number with 6 decimal places [3]

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Chapter 3

Floating Point Representation

Chapter Objectives

At the completion of this chapter, you would have learnt how to:

- perform floating point arithmetic;
- perform number storage.

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3.1 Introduction

When working with a single-length 12-bit word size we can cope with integral values in the range of -2048 to +2047. Similar limitation are imposed upon mixed or fractional values. However, we do have the option of extending these ranges by the use of double-length words but this is not always convenient.

A better alternative to fixed-point representation is the floating-point representation. It is capable of holding, in a single-length word, a greater range of numbers. It also uses the same form for coping with integers mixed numbers or fractions but at a cost of reduced accuracy.

Generally, the floating-point form is suitable for handling quantities of far higher values than usual, or whose values are every small.

3.2 Fixed Point and Floating Point Binary

- For fixed point binary the binary places are fixed.

  Example :

  0000.0000 (4 bits is reserved for integer part and the other 4 bits is reserved for fraction part)

- Using such representation the numbers that can be represented are limited.
- Example, if 01110.11 is to be stored using the above format only 1110.1100 could be reflected (note that most significant bit is a sign bit). To be more precise in representing number the floating point format is used.
- Using Floating Point 01110.11 could be represented as follow:

  $01110.11 = 0.1110110 \times 2^{100}$ (Using 8 bits to store number. The power is 100 or 4 in denary as four decimal places are shifted)

- Floating Point numbers are made up of three parts
  - Mantissa (e.g. 0.1110110_2)
  - Base (Radix) (e.g. 2)
  - Exponent (Power) (e.g. 100_2)

3.3 Floating Point Storage

$m \times B^E$ where

- Mantissa can be in
  - Sign modulus
  - 2's complement

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- Exponent can be in
  - 2's complements
  - Excess 2^{n-1} where n is the no. of storage bits for exponent

3.4 Floating Point Storage

Storing numbers in normalized floating point format can be defined with the following steps

Step	
1	Convert given numbers into binary
2	Check if converted is normalized, if not normalized it
3	Store mantissa based on given format (which can be sign modulus or 2's complement)
4	Store exponent based on given format (which can be 2's complements or excess 2^{n-1} method)

3.4.1 Store Floating Point Format

Format A

- Sign modulus mantissa and 2's complement exponent
  - Floating point numbers are stored using 16 bits. The first bit is the mantissa sign bit, next 9 are the normalized mantissa and the final six bits are the exponents in 2's complement.

Format B

- 2's complement's mantissa and 2's complement exponent
  - Floating point numbers are stored using 16 bits. The first ten bit 2's complement mantissa followed by a six-bit 2's complement exponent.

Format C

- Sign modulus mantissa and excess 2^{n-1} method
  - The first bit is the mantissa sign bit, the next 9 bits are normalized mantissa, the final six bits the exponent in excess 2^{n-1} form.

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Format D

- 2's complement's mantissa and excess 2^{n-1} method
  - Given 16 bits storage. First 10 bits 2's complement mantissa followed by 6 bits exponent in excess 2^{n-1} form.

Note: i. For mantissa stored using sign modulus form, 1 bit will be reserved to represent the sign, the absolute value of mantissa is stored without including the first bit before the decimal point.

ii. For mantissa stored using the 2's complement method, the first bit before the decimal point will be stored as it indicates the sign of the mantissa.

3.5 Floating Point Notation

- Scientific notation (decimal)

$$57429 = 5.7429 \times 10^{4}$$

Mantissa, Exponent

- Mantissa always 1 or greater, but less than 10

$$1 \leq |M| < 10$$

- Exponent always integer

Note: Mantissa can also be negative

3.6 Normalized Floating Point Form (Decimal)

- Divide mantissa by 10, increase exponent by 1
- Mantissa now always fraction, 0.1 or bigger

$$0.1 \leq |M| < 1$$

Example: 0.93×10^{8}, 0.9×10^{-28}

- Normalization gets rid of mixed numbers
  - Better for computers

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3.7 Normalized Exponent Form (Binary)

- Same principles as decimal
- Mantissa always fraction 0.1 (binary) or bigger

$$0.1 \leq |M| < 1$$

- Exponent is positive or negative integer

Note: Mantissa needs a sign bit but doesn't use two's complement

3.8 Storing Negative Mantissa

The mantissa can either be positive or negative. There are two alterative conventions used for storing the mantissa:

- Sign Bit
  - '0' is assigned if the mantissa is positive;
  - '1' is assigned if the mantissa is negative.
- Two's Complement Method
  - The two's complement of the mantissa is used if it is negative.

3.9 Storing Negative Exponent

Two alternative conventions also exist with respect to the storage of the exponent:

- Two's complement form
  - The exponent is stored in its two's complement form if it is negative and so we do not need to allocate a separate space to hold the sign of the exponent. Although we must always do so for that of the mantissa.
- Excess 2^{n-1} code of form
  (where **n** is the number of bits assigned for the exponent)
  - In this method, the value of 2^{n-1} is added to the actual exponent whether positive or negative to give the stored exponent.

$$\text{Stored Exponent} = \text{True Exponent} + 2^{n-1}$$

OR

$$\text{True Exponent} = \text{Store Exponent} - 2^{n-1}$$

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Exercises

1. **Express the decimal number +20.25 in sign modulus mantissa and 2's complement exponent format.**

Solution:

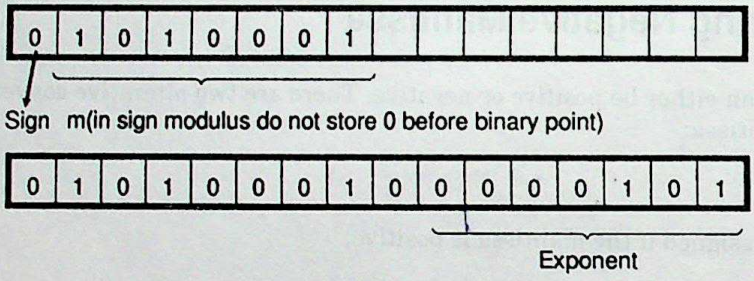
Steps:

i. $20.25 = 20 + 0.25$

$= 10100 + 0.01$

ii. $= 10100.01$

$= 0.1010001 \times 2^5$



Note: Format A Question

2. **Express decimal number +20.25 in 2's complement mantissa and 2's complement exponent.**

Solution:

Steps:

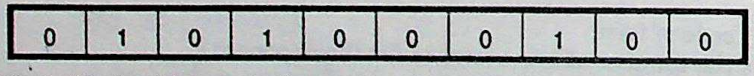
i. $20.25 = 20 + 0.25$

$= 10100 + 0.01$

ii. $= 0.1010001 \times 2^5$

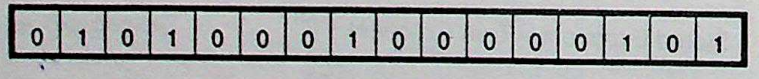
m

iii.



(note 0 before binary point is stored, an indicator of sign of mantissa)

iv.



Note: Format B Question

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3. **Express -20.25 in 2's complement mantissa and 2's complement exponent format.**

Solution:

Steps:

ii. $-20.25 = -(0.1010001) \times 2^5$

iii. ***Note:*** For negative mantissa in 2's comp sys, the way of representation is the same as negative integer that is

i. Invert all bits $0 \rightarrow 1$
$1 \rightarrow 0$

ii. Add 1

Thus

0	1	0	1	0	0	0	1	0	0
1	0	1	0	1	1	1	0	1	1
+									1

1	0	1	0	1	1	1	1	0	0

m

iv.

1	0	1	0	1	1	1	1	0	0	0	0	0	1	0	1

Note: Format B Question

4. Express -0.375 decimal in this format.

5. Express 20.25 in sign modulus mantissa and excess 2^{n-1} method format.

Solution:

Steps:

ii. $20.25 = \underbrace{0.1010001 \times 2^5}_{M}$

iii.

0	1	0	1	0	0	0	1	0	0						

Sign

iv. To store exponent in 2's excess method

4.1 Find no of bits for exponent in the case 6

4.2 Add excess 2^{n-1} to actual exponent
$2^{6-1} = 2^5 = 32$

4.3 Store exponent 32 + 5 = 37

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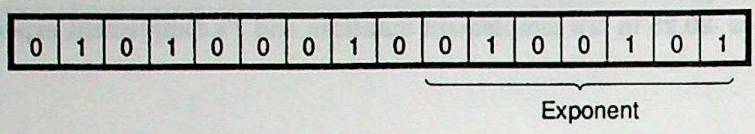

Note: Format C Question

6. **Express -0.375 in sign modulus mantissa and excess 2^{n-1} method format.**

Solution:

Step:

i. To convert given 0's and 1's in particular format, express string of 0's and 1's in $m \times 2^E$ form .

ii. Convert $m \times 2^E$ to $m1 \times 2^0 = m1$ (binary)

or

iii. Convert m1 to decimal

7. **Express the floating point number 0.110000000.011111 in decimal**

(***Note:*** Format C Question)

Solution:

$$0 \underbrace{110000000}_{m} \underbrace{011111}_{e}$$

Step

i. $0.11 \times 2^{31-32} = 0.11 \times 2^{-1}$

ii. $0.11 \times 2^{-1} = 0.011$

iii. $0.011 = \frac{1}{4} + \frac{1}{8} = 0.375$

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Past Years Questions

1. A floating point number is stored in normalized form using a ten-bit TWO's complement mantissa followed by a six-bit excess 32 exponent.
   a. Express 39.41634 in this form. [4]
   b. Express 0111011100 011101 as a decimal number. [4]

2. An 8 bits register is used to store value in excess form.
   a. Show how 14 is stored in it. [2]
   b. Calculate the true value from 10101011 which is in excess form. [2]

3. The number 25.828125 is to be stored using a 24 bit word floating point format. First bit is reservedfor sign bit for mantissa, the next 8 bits for exponent in Excess form and the rest for Normalized mantissa.
   a. Show how the number would be stored [5]
   b. Convert 010000100101101000000000 into denary [4]

4. Five bits of computer cell are allocated to store exponent in TWO's complement form.
   a. Show how exponent -12 is store in it. [2]
   b. Derive the true value from the stored number, in TWO's complement form, as shown below: [2]

1	0	1	0	1

5. A normalized floating point number uses a 10-bit TWO's complement mantissa followed by a 6-bit exponent in excess-32 form.
   a. Store the denary number -43.123 into the above word format [5]
   b. Convert the following binary word from the above format into denary [5]

   101 001 001 0 101 00

6. A normalized floating point number, using a 10 bit TWO's complement mantissa followed by a 6-bit exponent in TWO's complement.
   a. Store the denary number 68.25 into the above word [4]
   b. Store the denary number -0.1267 into the above word [6]

7. The number -17.546875 is to be stored in floating point format is a SIXTEEN-BIT word. ONE bit is to be used for the sign of the mantissa, the next FIVE bits for the exponent in TWO's COMPLEMENT and TEN bits for the NORMALIZED mantissa. Show how the number would be stored. [7]

8. a. Show how 23.7 is held in fixed point form with 4 bits after the point [2]
   b. What DENARY value is actually held? [1]

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9. In a SIXTEEN bit register, numbers are stored in NORMALIZED, FLOATING-POINT form i.e. MANTISSA (first 10 bits) then EXPONENT (last 6 bits). Each part is in TWO's COMPLEMENT form, with the binary point assumed to be AFTER the LEFT HAND binary digit of the mantissa.

a. What DENARY number is represented 1001011010001001? [3]

b. Express the denary number, -23.875 in this format. [4]

10. A floating point number is stored in normalized form using a 16-bit word, with the bit reserved as sign for the mantissa, the next 9 bits reserved for the mantissa itself, and the remaining 6 bits reserved for the exponent in excess 32 form.

Express -36.4515 in this form. [4]

11. A floating point number is stored in normalized form using a 16-bit word, with the first bit reserved as thesign for the mantissa, and the last 6 bits reserved for the exponent in TWO's complement. The remaining bits are for the mantissa itself.

a. Express -16.875 in this form [4]

b. Express -0.09375 in this form [4]

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Chapter 4

Representation Of Instruction

Chapter Objectives

At the completion of this chapter, you should be able to:

- understand instruction word format;
- describe the differences between addressing techniques;
- have an understanding of instruction types.

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4.1 Introduction

The principles of architecture extend through almost every aspect of computer organization. Included are the lengths of the instruction words, whether or not length is variable, and how many addresses in memory are referenced by an instruction word.

Other architectural considerations concern the number of bits in each memory word, whether numbers are handled in 1's or 2's complement form or in BCD or some combination of these. What are the instructions provided, how are the memories organized, and how are input/output devices interfaced? As can be seen, computer architecture is a large and rich subject that deals with most aspects of computer design and organization.

4.2 Construction of an Instruction Word

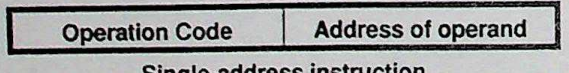
A computer word is an ordered set of characters handled as a group. Basically all words consist of a set of binary digits, and the meaning of the digits depends on several factors. For instance, the bits 01000100 could represent the decimal number 68 in a pure binary computer, and the decimal number 44 in a BCD computer that uses an 8, 4, 2, 1 code. Thus, the meaning of a set of digits is determined by its usage.

Other interpretations are possible, for instruction words are stored just as data words are, and the digits could represent an instruction to the computer. Since memory locations can store either instruction words or data words, the programmers and system operators must see that the instruction words are used to determine the sequence of operations that the computer performs, and that the computer performs, and that reasonable meanings are assigned to the words.

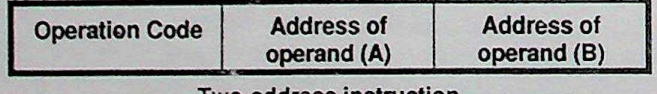
4.3 Instruction Word Formats

4.3.1 Numbers of Addresses

The number of divisions in the basic instruction word is determined primarily by the number of addresses refered to. The single-address instruction has been covered. Most current computers however, have two-address instruction words with three sections; the first section consists of the op code and the second and third sections generally each contain the address of an operand. Different computers used addresses differently.



Single-address instruction



Two-address instruction

In many computers, instead of a single accumulator, there are two or more registers called either multiple or general-purpose registers, or simple general registers. Instruction words with two addresses can operate in one of the following modes:

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- Memory, memory. Both addresses refer to memory locations. For two memory address A and B, the instruction ADD A, B would cause the operation.

$$MEM[A] \leftarrow MEM[A] + MEM[B]$$

Note: MEM[A] means "The contents of address A in memory".

Example: Add the numbers in memory locations 963 and 492, and store into memory location 963.

Add	963	492
	MEM	MEM

- **General register, memory.** Address A in the instruction word specifies the general register to be used, and the second address indicated the location in memory. A LOAD ACCUMULATOR instruction (LOAD A, B) would cause the contents at location B in memory to be transferred into the general register denoted A. In symbols, $GR_A \leftarrow MEM[B]$.

Example: Add the number at memory location 492 into the operand in GR_3 and store into GR_3.

Add	3	492
	GR	MEM

- Memory, general register. The memory address A gives the location of the operand in memory, and the general register code B is the register involved. An instruction might be a STORE instruction (STO A, B), which would move the contents of the general register into memory: $MEM[A] \leftarrow GR_B$

Example: Add the value in GR_4 into the value address at 1738 and store into address 1738.

Add	1738	4
	MEM	GR

4.4 Representation of Instruction and Data

Important features of a computer's architecture are the number of bits in instruction words, the size of memory words, and the way data are represented in the computer.

Business data processing involves much manipulation involving character strings (names, addresses, text, etc.). The desire to create computer architectures that conserve on instruction word length and also permit efficient storage of character strings of arbitrary length has led to a number of computer architectures with:

- only 8 bits at each address in memory, so that a single alphanumeric character an be stored at each address, and

- instruction words with variable lengths (each word length is some multiple of 8 bits)

As a result, most computers now have memories where addresses are 8-bits bytes. Instruction words are variable length, with each being some multiple of 8 bits. Data words are also multiples of 8 bits, with many computers having 8-, 16-, 32-, and even 64-bit data word lengths.

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4.5 Addressing Techniques

There are several techniques used to specify a memory address. The use of any of these techniques is generally motivated by one of the following considerations:

- Desire to shorten address section
- Programmer convenience
- System operation facilities

The basic addressing techniques now in use: direct addressing, indirect addressing and indexed addressing.

4.5.1 Direct Addressing

Simply giving the complete binary memory address is the most direct way to locate an operand or to jump. As a result, most computers provide for some form of direct addressing.

Example : ***MOV AX, TABLE***

This instruction loads the contents of data memory location TABLE
into the AX register.

4.5.2 Indirect Addressing

Another widely used addressing variation is called indirect addressing where the instruction word gives the address not of the operand to be used, but the address of the address of the operand.

For example, if we ADD 302 and the instruction is a conventional direct-addressing instruction, the number at location 302 will be added to the word currently in the accumulator. If the addition instruction is indirectly addressed, and we write IAD 302 (indirect ADD), then the number stored at address 302 will give the address of the operand to be used. As an example, when the instruction word at address 5 in the following memory is performed, it will cause the number 164 to be added to the current contents of the accumulator.

Memory	Contents
5	IAD 302
:	:
:	:
302	495
:	:
:	:
495	164

4.5.3 Indexed Addressing

- There is a variation on conventional direct memory addressing that facilities programming, particularly the programming of instruction sequences that are to be repeated many times on a sets of data distributed throughout the machine. This technique is called indexing.

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4.6 Instruction Types

The instruction set can be classified under 4 main groups:

- Data Transfer
- Arithmetic
- Control Transfer
- String Instructions

4.6.1 Data Transfer Instructions

Data transfer instructions move data and addresses between register and memory location or I/O ports.

Example : ***Move [MOV]***
The general form is : ***MOV destination, source***

The fundamental general-purpose instruction is move[MOV]. It can transfer byte or word data between register and a memory location or between two registers. The source can also be a numeric value. It can also transfer an immediate data value into a register or memory location.

However, the following must be noted:

- data cannot be moved between two memory locations directly.
- cannot load an immediate value. E.g. load the offset address of a segment into a segment register.

4.6.2 Arithmetic Instructions

These instruction performs arithmetic operations on binary or BCD numbers. Three are instructions for the four standard arithmetic functions namely:

- addition
- subtraction
- multiplication
- division
- logical

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4.6.3 Control Transfer

These are instructions that can transfer program execution from one part of memory to another. These instructions can be divided into 3 groups:

- Unconditional transfers
- Conditional transfers
- Iteration controls

4.6.3.1 Unconditional Transfers

- It is generally used to skip over a group of instructions that executed from some other part of the program.
- Example: Call Procedure-name.

4.6.3.2 Conditional Transfer

- In the conditional Transfer instructions, if the condition is satisfied, the instruction will be executed, otherwise it will continue on to the next instruction in the program.
- All conditional transfer instructions occupy two bytes in memory. The first byte holds the operations code and the second bytes holds the relative displacement.

4.6.3.3 Iteration Controls

- The general form is ***LOOP target***.
- The iteration control instructions are conditional transfer, but have a very special purpose. They set up repetitive loops.

4.6.4 String Instructions

- The string instructions enable us to operate on blocks of bytes or words in memory. These blocks, or strings, may be up to 64K bytes long, and may consist of numeric values (either binary or BCD), alphanumeric values (such as ASCII text characters) or, for that matter, any kind of information that can be stored in memory as binary patterns.

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Exercises

1. Briefly describe what is direct, indirect and indexed addressing.

2. Describe the difference between direct and indirect addressing.

3. Briefly describe three instruction types.

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Chapter 5

Errors And Accuracy

Chapter Objectives

At the completion of this chapter, you should be able to:

- identify different types of errors;
- calculate absolute and relative error;
- estimate maximum absolute error:
- generate check digit.

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5.1 Introduction

In building a system, measures must be taken against possible human mistakes. Procedures must be set up to help people to detect any mistakes. The role of data preparation or data entry are prone to errors, therefore they must not be overlooked as the ill effect may affect the entire system seriously.

5.2 Mistakes

At the man/machine input interface, i.e. when a person want to enter data into a computer system. The common *mistakes* are

- ❒ Lose track of the source documents
- ❒ Enter data wrongly
- ❒ Invalid datatype
  e.g. enter alphabets instead of numeric data
- ❒ Out of the valid data range
  e.g. enter 102 while the valid range is 1 to 100
- ❒ Transposition of characters
  e.g. enter 23322 while the actual data is 23223
- ❒ Interference in data transmission from a remote site.
  e.g. 00100101 is transmitted along a cable and on the way due to voltage surge, the data becomes 10100101

and many more.

Mistakes are due to human carelessness.

- ❒ Parity bit check

  Example:

  If 'B' is represented as 100 0010, we can use *even parity conversion* to produce a parity bit.

```
IF odd number of 1's THEN
        parity bit    ← 1
    ELSE
        parity bit    ← 0
ENDIF
```

  The parity bit may be appended to the left of the 100 0010, 'B' is represented as 0100 0010. Suppose the data is received as 0110 0010, there are three 1's which should give 1 in the parity bit, then the mistake is detected.

- ❒ Check digit method
  - A Check digit is derived from the true data before transmission, which is then appended to the right of the true data for validation.

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Step 1: A true data 445, each of the digit is assigned a weight,

```
4  3   2  ←—— Weight age
4  4   5

5 * 2 = 10
4 * 3 = 12
4 * 4 = 16
      38
```

Step 2: The sum of 38 is to be divided by a modulus
(In this example, 11 is used as it can detect most of the mistakes).

38/11 gives quotient 3 and the remainder 5.

Step 3: Remainder 5 is subtracted from the modulus (11) to obtain the check digit '6'.

11 - 5 = 6

Step 4: The check digit can be appended to the right of 445, to give 4456, which could be validated.

In order to verify the correctness of 4456, we could just multiply 4456 by the set of weight, 6 is assigned the weight of 1,

```
4  3   2   1←— weight age
4  4   5   6
(4*4) + (4*3) + (5*2) + (6*1) = 44
```

44 is divisible by 11, thus conclude that 4456 is received correctly.

It has been tested that MOD 11 is most efficient in detecting mistakes, reason being 11 is a prime number. Though bigger number is better, the small increase in efficiency is overwhelmed by large increase in processing.

Check point

1. List five types of mistakes which may occur during the data preparation and data entry stages of the processing cycle.

2. Give one other example each of the appropriate use of the hash and batch totals. Describe the principle of a parity check and how even and odd parity differ.

3. Using mod 7 and weighting 2,3,4,5.... append appropriate check digits to:

   a. 439
   b. 1058535

   Using the same mod 7, determine whether the following are valid:

   c. 1794
   d. 10545722

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5.3 Errors

5.3.1 Inherent Error

It is error that already exist by itself in the measurement scale. E.g. Take a ruler of measurement, though there are many small divisions, there is still the smallest division that is not sub-divided further. Thus there is uncertainty in reading the length as mentioned above.

If the length fall between 5.1cm and 5.2cm, the smallest division is 0.1cm, then the reported reading may be 5.15 cm and the *inherent error* is 0.05cm.

5.3.2 Induced Error

It is the error which is brought in from 'outside' due to external factors.

Example: In a fixed point 8-bit register in computer, the implied point is to the right of 5 bits.

If a data 1010.01110 is to be stored, rounding or truncation is necessary. Rounding

The above data is stored as

0	1	0	1	0	1	0	0

Thus the error is brought in due to the limitation of computer.

Check point

4. What is the difference between error and mistakes?

5.4 Measurement of Error

- Absolute error
  - If a true value 2.5 is reported to be 3 (round to full number) an error of 0.5 is induced. This error is called the absolute error.
  - Absolute error
    E_a = | reported value - true value |

    | | means the absolute value is taken, the sign is not considered at all.
- Relative error
  - Another more meaningful measurement of error is relative error, because it expresses the absolute error as a proportion of the true value (if the true value is not available, the reported value is used.)

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- Relative error

E_r = |absolute error / True Value|
= |absolute error / True Value| * 100%

Maximum relative error between any TWO values reported as
R = 100 * (error bound/R)

Example: 3.6 is rounded to 1D, the Maximum error between 3.55 and 3.65 = 100 * (0.1/3.6) = 2.78 %

Check point

5. What is the error bound of the values reported as:

   a. 3.5cm
   b. 45.3 seconds

6. Calculate the maximum relative error on:

   a. 13.5
   b. 130.5
   c. 1300.5

5.5 Error Propagation

Inherent errors in data values, may produce further errors as they are operated on arithmetically, these errors may be termed as Induced errors. This spread of errors is called the Error Propagation.

❒ Addition

Example: 5+7, each of the data values are rounded to whole number.
5.5 + 7.5 = 13 Max
4.5 + 6.5 = 11 Min

E_r = 100 * (1/12) = 8.33%
E_r of 5 = 10% 0.5/5 x100
E_r of 7 = 7.1 % 0.5/7 x100

You may try 13 + 6, a conclusion may be derived.

Conclusion: Relative error of the sum is between the relative error of the individual reported values.

E_a = absolute error of
$E_a(A + B) = E_a(A) + E_a(B)$

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- Subtraction

Example: 8-4, each rounded to whole number.

8.5 - 3.5 = 5 Max
7.5 - 4.5 = 3 Min

$E_a(8-4) = 1 = E_a(8) + Ea(4)$
$E_r = 100 * (1/4) = 25\%$
E_r of 8 = 100 * (0.5/8) = 6.25%
E_r of 4 = 100 * (0.5/4) = 12.5%

You may try 3-2

Conclusion: Relative error of the subtraction is more than the relative error of the individual reported values.

$E_a(A-B) = E_a(A) + E_a(B)$

- Multiplication

Example: 3*4*5, each rounded to whole number.
3*4*5 = 60,
3.5*4.5*5.5=86.625, Max
2.5*3.5*4.5=39.375, Min

Error bound = 86.625 - 39.375 = 47.25

This error bound is very big indeed, and it is not symmetrical about 60 (the result of multiplying 3,4 and 5). If we quantify the relative error, it would be as follow:

Take half of the error bound,
$E_r = 100 * (23.625/60) = 39.375\%$
E_r of 3 = 100 * (0.5/3) =16.667%
E_r of 4 = 100 * (0.5/4) =12.5%
E_r of 5 = 100 * (0.5/5) =10%

You may try 3*3*3, what conclusion you may draw?

Conclusion: Relative error of the multiplication is approximately the sum of the relative errors of the individual reported values.

$E_r(A*B*C) = E_a(A)/A + E_a(B)/B + E_a(C)/C$

Induced error in multiplication is the sum of the inherent error in each reported value.

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❒ Division

Example: 7.5/5.3, each is rounded to 1D

7.55/5.25 = 1.438, Max
7.45/5.35 = 1.393, Min

Error bound = 1.438 - 1.393 = 0.045
E_a = 0.045/2 = 0.0225

E_r of (7.5/5.3) = 100 * (0.0225/1.415) = 1.59%
E_r of 7.5 = 100 * (0.05/7.5) = 0.667%
E_r of 5.3 = 100 * (0.05/5.3) = 0.943%

Try 3.5/1.5,

Conclusion: Relative error of the division is approximately the sum of the relative errors of individual reported values.

$$E_r(A/B) = E_a(A)/A + E_a(B)/B$$

Finally, try a combination of arithmetic operation, multiplying and division.

Example: (3.5*13.5*1.5)/(7.0*4.5*40.5)

In what ways the error propagated is maximized?
Multiply the top row and bottom row respectively, then divide the numerator by the denominator.

In what ways the error may be minimized?
(3.5/7.0) * (13.5/4.5) * (1.5/40.5), the individual divisions will produce values which are relatively small, overflow error may be reduced.

❒ Rounding

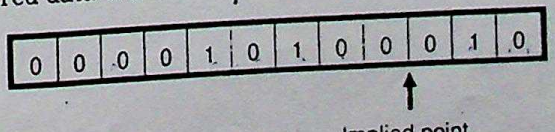
- Rounding errors frequently occur when doing manual decimal arithmetic. They occur with even greater frequency in computer arithmetic.

Example: If 0.0056 is rounded to 3 decimal places the answer is
A rounding error of 0.006 - 0.0056 = 0.0004 is therefore induced.

If 0.0055 is rounded to 3 decimal places, we may use the following rule:
If the third decimal place is odd, do nothing;
else add 1 to the third decimal place.
0.0055 is rounded to 0.005.
If 0.0045 is rounded to 3 decimal places, the reported value is 0.005.

❒ Truncation

- If a storage space that can only accommodates 4 digits is used to store a number 67890, the least signification digit '0' will be chopped off giving 6789 as the number stored.
- If 1010.0010111 is to be stored in 12-bit register with 4 bits to the right of the implied point. The stored data will be



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❑ Overflow

- Overflow is different from truncation error in that the most signification digit will be drop-out instead of the least significant. Storing 67890 in 4 digits, the most significant digit '6' is chopped off.
- This error is very serious in computer programming, thus many compilers and interpreters are written such that they can deliver *'overflow error'* message.

❑ Significant figures

- At times, we can only take a certain number of significant figures. Take for example the population of Singapore, we may say there are 3,500,000 people taking only 2 significant figures. Even though we may have found the population to be 3,547,893, as the less significant digits 47,893 is small compared to 3,500,000.
- A real number 2.03452355 may be expressed in 5 significant figures, as 2.0345.

❑ Converting from one number system to another

- To convert 0.7 decimal to binary number will induce an error as shown below:

```
.7   *   2
1.4  *   2
0.8  *   2
1.2  *   2
0.4  *   2
0.8  *   2
1.6...
```

The series of multiplication is recurring, if we to stop at 7 binary places, we have to accept that an error is induced.

$$0.7 = (0.1011001)_2$$

Rounding, truncation, overflow, significant figures and change of number system will all lead to *induced* errors in data.

Check point

7. What is the effect on error if 3.5*7.25 is:

   a. Calculated to integer precision;
   b. Cut to 1D;
   c. Rounded to 1D;
   d. Calculated to 3 significant figures;
   e. Worked out 'in full', with no cut or rounding

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5.6 To Control Errors

We want to calculate the volume of a sphere of radius 5.5 cm, the formula below can be used:

$$V=(4/3)*\pi r3$$

Take $\pi = 3.141$, and $r = 5.5$

		Er	
r	5.5	0.91%	Inherent
V	696.7785	3*0.91 = 2.73%	Induced

This means that V has an absolute error of (2.73/100) * 696.7785 = 19.02

This means that range of Volume is 696 ± 19 cubic centimetres.
The decimal places are not significant.

To reduce the induced error, we should, if possible, take 'r' to 5D and do the same for π. Raise 'r' to the third power, multiply by n, multiply by 4 and finally divide by 3. Only by taking such precautions, could we minimize the errors.

Answers to check points

1. There are many possible sources of mistakes at this stage;
   Document omitted or misread;
   Transposition errors;
   Characters omitted;
   Mis-keying;
   Out of range;
   Invalid datatype;

2. There are many possible answers:
   Hashing the invoice numbers;
   Hashing the dates;
   Batching totals of amount of money;
   Batching totals of number of items withdrawn from store;
   Parity checking is testing the make-up of the binary digits in a coded eight-bit byte.

   Even parity: by making the leftmost digit a 1,
   if the total numbers of 1's in the remaining seven bits is even;

   Odd parity: by making the leftmost digit a 1,
   if total number of 1's in the remaining seven bits is odd;

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3.

Data	Hash Total	Divide by 7, take remainder	7-remainder	Final data
439	43	1	6	4396
1058535	117	5	2	10585352

Data	Hash Total	Divide by 7, remainder	
1794	47	5	not valid
10545722	105	0	valid

4. Mistakes are made due to human carelessness in handling data. Errors arise from the data itself, i.e. Inherent error; it may be brought in due to external factors like rounding, limit of computer systems, i.e. Induced error.

5. Error bound = 3.45cm -- 3.55cm

Error bound = 45.25s -- 45.35s

6.

Data	Relative error
13.5	0.37%
130.5	0.038%
1300.5	0.0038%

7. The reported answer is 25.375

a. 3*7 = 21 Relative error = 17.2%

b. 25.2 Relative error = 0.7%

c. 3.5*7.3=25.55, Relative error = 0.7%

d. 25.375 = 25.4 (3 sig.f.) Relative error = 0.1%

e. 3.55*7.255 = 25.75525, Max
3.45*7.245 = 24.99525, Min
Error bound = 0.76
Relative error = 100*(0.38/25.375) = 1.5%

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Points to Remember

- ❒ To distinguish mistake from error.
- ❒ To be aware of how to reduce mistakes.
  The methods are *hash total, batch total, parity-bit method* and *check digit method.*
- ❒ Inherent error
  - It is error that exist by itself in the measurement scale.
- ❒ Induced error
  - It is error that is brought in from 'outside' by external factors. Usually, induced error is the result of propagation of inherent errors.
- ❒ To measure error using two quantities
  - Absolute error E_a = $\bar{\rho}\varepsilon\pi o\rho\tau\varepsilon\delta$ value - true value'
  - Relative error $E_r = E_a$/True value
- ❒ Error can spread, or propagate
  - $E_a(A+B) = E_a(A) + E_a(B)$

    As a result of addition, the absolute error of (A+B) is the sum of absolute error of A and that of B.
  - $E_a(A-B) = E_a(A) + E_a(B)$
  - $E_r(A*B*C) = E_a(A)/A + E_a(B)/B + Ea(C)/C$
  - $E_r(A/B) = E_a(A)/A + E_a(B)/B$
- ❒ Types of errors
  - Rounding; Truncation; Overflow; Conversion
- ❒ Divide a number by another, before multiplying them, is a method to reduce possible overflow error.

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Past Year Questions

1. Given that X = 5.63 and Y = 7.82, where each number has been rounded to 2 decimal places, calculate to 4 significant figures.
   a. Relative error when they are added [2]
   b. Relative error for X * Y [2]

2. x = 0.34 and y = 0.55, both corrected to 2 decimal places.
   Find
   a. the absolute error of 2x + y [2]
   b. the relative error of 2x + y [2]

3. Two values are recorded as 8.7 and -4.3, both correct to 1D.
   What is the ABSOLUTE ERROR when they are added together? [2]

4. Calculate the absolute and relative error of a + b given a = 3.5 (corrected to ID) and b = 1.55 (corrected to 2D). [4]

5. Pick the correct completion for each of the following sentences: [3]
   a. Discarding least significant digits of a number so it will fit in a fixed-size storage location [1]
      i. is a method of implementing floating-point numbers
      ii. is called truncation
      iii. does not happen on modern computers
   b. The difference between the reported value and the true value of a quantity [1]
      i. is the absolute error
      ii. is zero
      iii. is the inherent error
   c. Error can arise in computations [1]
      i. only when the initial readings for values are inaccurate
      ii. if the mantissa of a computed result won't fit in the storage space allocated for it without loss of data.
      iii. because no real numbers can be represented exactly in a floating-point representation.

6. _____ errors are ones which reflect the smallest measurement possible one the scale used [1]

7. Subtraction is _____ susceptible to error propagation than is addition. [1]

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8. Explain briefly what is inherent and induced error. [4]

9. Using mod 11 and weighting of 2, 3 and 4 for units, tens, and hundreds columns, calculate a check digit for 834 [3]

10. An account number consists of 10 digits, the tenth digit being a CHECK DIGIT, MODULES 11.
    a. Calculate an appropriate check digit for 005611350. [4]
    b. Which of these numbers would be declared invalid:0765213141, 0265073147? [2]

11. Using mod 7 and weighting of 2, 3, 4 for the units, tens and hundreds columns append an appropriate CHECK DIGIT to each of 928 and 921. [3]

12. Using modules 11 and weighting of 2, 3 and 4 for the units, tens and hundreds columns:
    a. append a CHECK DIGITS to the number 758 [2]
    b. show why there is a problem when generating a check digit for 748, and suggest how to overcome this problem [2]
    c. determine whether or not 8752 is a VALID CODE [2]

13. a. Using mod 11 and weighting of 2, 3 and 4 for the units, tens and hundreds columns, append a check digit to 974. [2]
    b. Show how a transposition error occurring in the number 974 would be detected. [2]

14. a. What is parity in a parity check? [1]
    b. When will a parity check fail? [1]

15. a. How is data verification performed? [2]
    b. Why might data verification be used? [1]
    c. How might the data entry be validated? [2]

16. Describe what is the purpose of a check digit? [2]

17. a. Write the decimal fraction 4.0 as binary fraction using only four binary digits after the decimal point. [1]
    b. Convert your answer back to denary, and determine the absolute error. [1]
    c. Calculate the relative error. [2]

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Chapter 6

Set Notation / Representation And Probability

Chapter Objectives

At the completion of this chapter, you should understand:

- the logical requirements of a set;
- how to enumerate and specify sets;
- the use of Venn diagrams;
- problem solving;
- relations between sets;
- that in real life problems, we have to deal with likelihood and not 100% certainty. Probability model allows us to evaluate the chances or likelihood;
- probability and know the valid range of values of a probability;
- Venn Diagram as probability spaced;
- and find probability of combined events;
- and use tree diagrams to depict all possible outcomes in a program, it helps in determining the probability of relevant event.

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6.1 Introduction

Many real life problems can be analysed further as some collection of objects, each collection of objects may have logical relationship with another. This chapter will tell us how sets and Venn diagrams can be used to real life problems.

6.2 Definition of Set

A set is a collection of objects, things or symbols, for instance, numbers. names of people, colors etc.

The individual objects in a set are called elements or members of the set. All the members of the set have a common feature, which links them in some way.

The elements in a set must be unique.

6.3 Notation

To identify specific sets, we may either give them names, or just using capital letters.

For example :

A={2,4,6,8,10} or {x:x is even integer and $1 \leq x \leq 10$}

B={1,3,5,7,9} or {x:x is odd integer and $1 \leq x \leq 10$}

C={CAT,DOG,HORSE,LION,RABBIT,TIGER}

D={BLACK, BLUE, GREEN, ORANGE, PINK, RED, YELLOW}

E={x:x is a month in the year}

Take set E as an example, we know that July is a member of E, while Tuesday is not a member of E. We therefore abbreviate the phrase "is a member of" by using the Greek letter epsilon, $\in$, and $\notin$, to denote "is not a member of".

As for the above examples, we may write CAT $\in$ C, July $\in$ E, but Tuesday $\notin$ E.

6.4 Venn Diagrams

We have learn two methods of expressing a set:

- By listing the elements within brackets, {}.
- By specifying the main characteristics of the set.

Beside these two ways, we can also use diagram, Venn Diagram. Usually a rectangle is used to denote the universal set U.

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For example, U={1,2,3,4,5}, A={1,2} and B ={4,5} then the Venn Diagram is

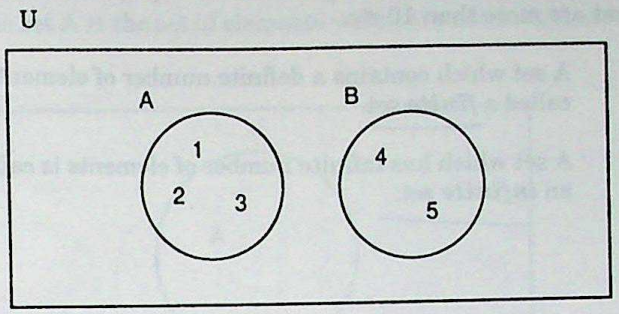

Figure 6-1

6.5 Relation Between Sets

6.5.1 Universal Set

For all problems, there exists a fixed set which called the Universal Set. This is the largest set which contains all the related items.

The Universal Set is denoted by U or ξ.

6.5.2 The Null or Empty Set

The sets that we discussed above contain at least one element. There are some sets that do not contain any element at all. We call them the null or empty set.

The Empty Set is denoted by ∅ or a pair of empty brackets {}.

Examples of null set are :

- The set of pupils in your class who are less than 12 years old
- The set of months with 12 days
- The set of cats in Singapore with four tails
- The set of cars with 68 doors

6.5.3 The Finite and Infinite Set

Question: How many elements are there in the set of English Letters?

Answer : 26 letters i.e. a..z

If we let E be the set of all English Letters, then the number of element in this set E is 26, and we write as n(E) = 26.

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In most of the cases, we are able to tell the number of elements in a set, but however, there might have cases which we cannot give an exact figure. For example, the set of even number, the set of prime number that are more than 10 etc.

Definition : A set which contains a definite number of elements is called a ***finite*** set.

A set which has infinite number of elements is called an ***infinite*** set.

6.5.4 Subset

A set P is a ***subset*** of a set Q if every elements of P is also element of Q, we write $P \subseteq Q$

Example:

$U = \{1,2,3,4,5,6,7,8,9,10,11,12\}$

$A = \{2,4,6,8,10\}$

$B = \{1,2,3,4,5,6,7,8,9,10\}$

So, $A \subseteq B \subseteq U$

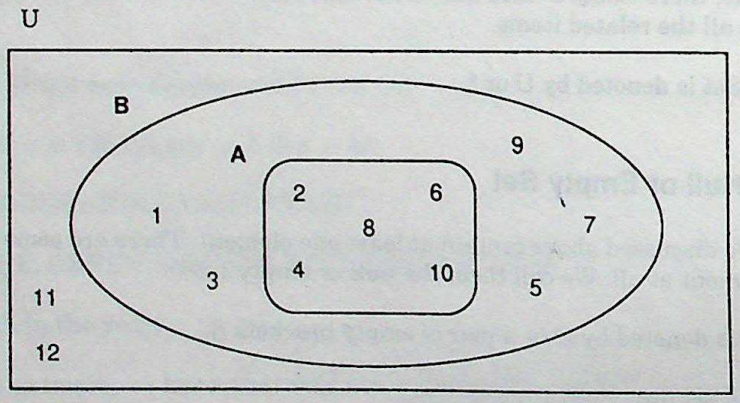

Figure 6-2

❒ Facts

- $A=B \Leftrightarrow A \subset B$ and $A \supset B$
- The empty set is a subset of every set.

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6.5.5 Complement

The ***complement*** of the set A is the set of elements which are not in A, denoted by $\overline{A}$.

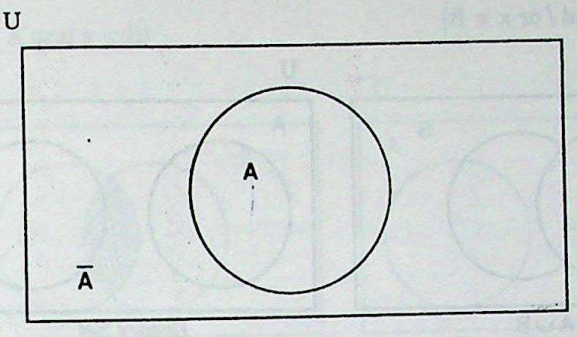

Figure 6-3

Examples:

1. If U = {1,2,3,4,5,6,7,8,9,10}, and A = {1,3,5,7}
   then $\overline{A}$ = {2,4,6,8,9,10}

2. If U is the set of real numbers, and A is the set of all negative real numbers then $\overline{A}$ is the set of all positive real numbers.

❒ Fact

- The complement of the universal set is the empty set.
- The complement of the empty set is the universal set.
- The complement of $\overline{A}$ is A

  i.e. $\overline{\overline{A}} = (\overline{\overline{A}}) = A$

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6.5.6 Union

The ***union*** of two sets, A and B, is the set of all elements of A and B we write.

i.e. $A \cup B = \{x : x \in A \text{ and / or } x \in B\}$

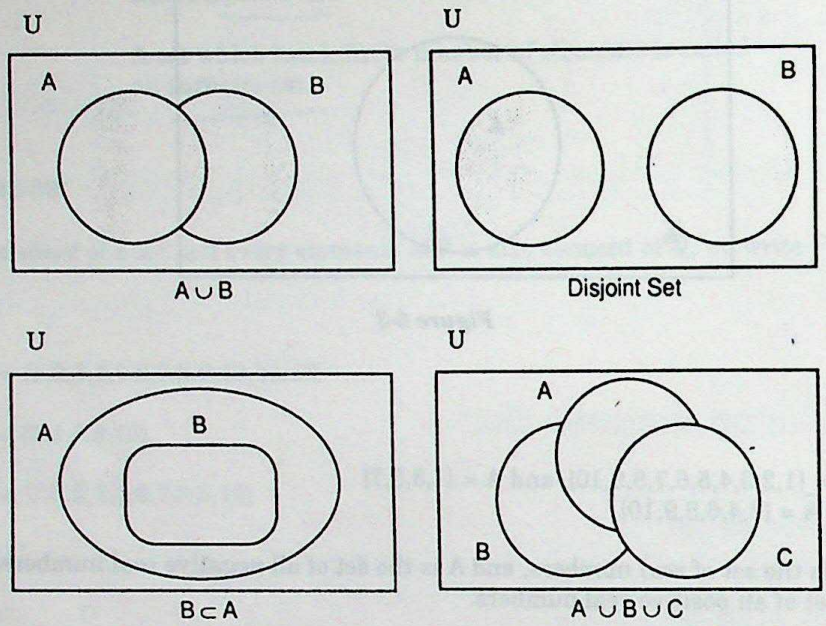

Figure 6-4

Example: If $A = \{1,2,3,4\}$ and $B = \{1,2,5,6\}$ then $A \cup B = \{1,2,3,4,5,6\}$

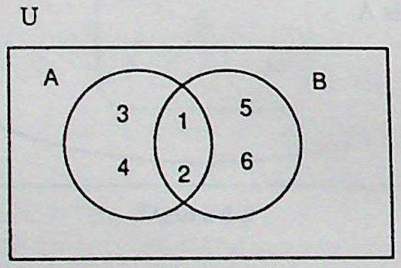

Figure 6-5

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6.5.7 Intersection

The intersection of two sets A and B is the elements which belong to both A and B, we write $A \cap B$.

i.e. $A \cap B = \{x : x \in A \text{ and } x \in B\}$

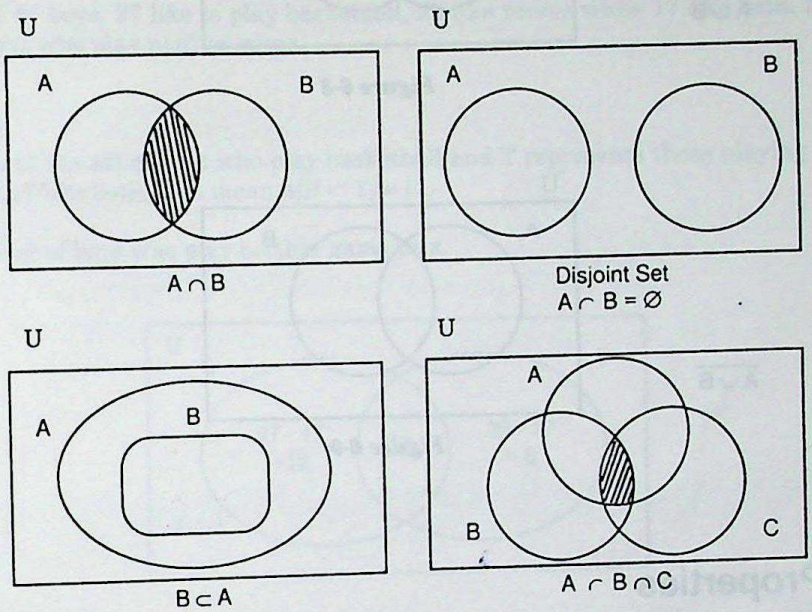


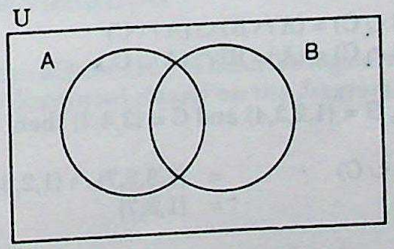
Figure 6-6

Example: If $A = \{1,2,3,4\}$ and $B = \{1,2,5,6\}$ then $A \cap B = \{1,2\}$

❒ Fact

- If $A \cap B = \varnothing$ then A and B are disjoint.

6.5.8 More Examples



$A \cap \overline{B}$

Figure 6-7

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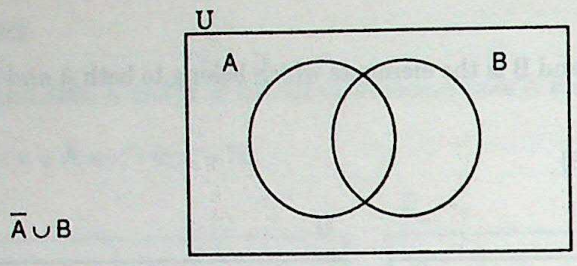

Figure 6-8

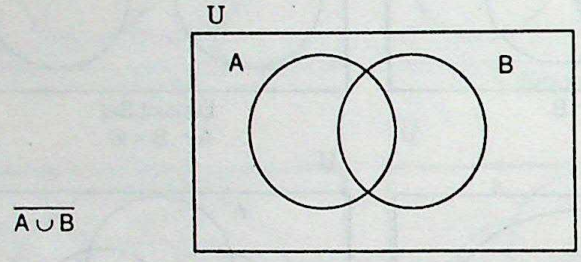

Figure 6-9

6.6 Properties

- Commutative Property

$$A \cap B = B \cap A$$
$$A \cup B = B \cup A$$

- Associative Property

$$(A \cap B) \cap C = A \cap (B \cap C)$$
$$(A \cup B) \cup C = A \cup (B \cup C)$$

- Distribution Property

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Example: If A = {1,3,5,7}, B = {1,2,3,4} and C = {3,4,7} then

i. $A \cap (B \cup C)$ = {1,3,5,7} ∩ {1,2,3,4,7}
= {1,3,7}

ii. $(A \cap B) \cup (A \cap C)$ = {1,3} ∪ {3,7}
= {1,3,7}

Prove the second *distribution property law.*

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6.7 Applications

Use of *Venn diagrams in Solving Problems*

Example:

In a group of 40 boys, 27 like to play basketball, 25 like tennis while 17 like both. Deduce the number of boys who play neither game.

Solution:

Let B represent the set of boys who play basketball and T represents those playing tennis. We are told that 17 like both, that mean $n(B \cap T) = 17$.

Let the number of boys who play neither game be x.

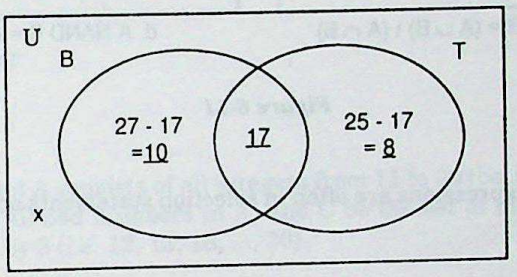

Figure 6-10

We are given n(U) = 40
therefore, 40 = 10 + 17 + 8 + x
= 35 + x
x = 40 - 35
= 5

6.8 Other Logical Relation

Venn diagrams in set theory can also be used to describe logical relationship. Namely AND, OR, XOR and NAND, they can be illustrated clearly on the diagram.

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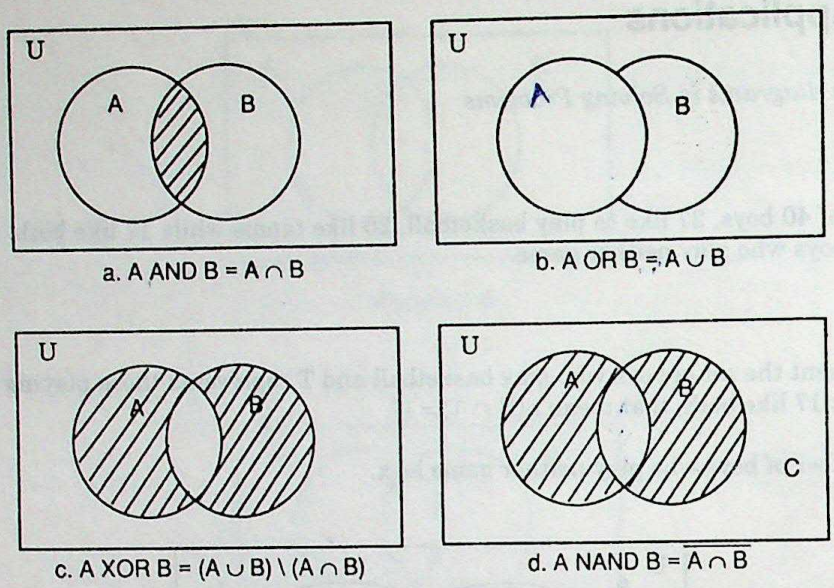

Figure 6-11

In programming, logical expressions are often in selection statements and repetition statement.

Example :

If Age >21 AND height >=1.70

Age >21, this condition will satisfy a set and height >=1.70 will form another set:
A = {Age : Age>21}
B = {height : height >= 1.70}

The operator AND, has produced a region that is the common region of both sets, as shown in Fig 6.11 a.

Points to Remember

- A set is defined as a collection of items which share a common feature and all items are unique.
- Use Venn diagram to represent various sets. It is a good model to solve some real life problem.
- Relationship between sets

Subset	$\subset$		
Intersection	$\cap$	$\Leftrightarrow$	AND
Union	$\cup$	$\Leftrightarrow$	OR
Complement	$\overline{A}$	$\Leftrightarrow$	NOT

$\Leftrightarrow$ corresponds to

AND, OR and NOT are logical operators available in many computer languages.

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Past Years Questions (Set Theory and Venn Diagram)

1. If the universal set P consists of all integers from 1 to 30 inclusive set Q consists of all prime numbers in the range 10 to 30 inclusive and set R contains all integers in the range 10 to 20 inclusive. Find :
   a. $(R \cap Q)$ [1]
   b. $(R \cup Q)'$ [3]

2. Draw Venn diagram for the following:
   a. $(A' \cup B) \cap (A' \cup C)$ [2]
   b. $(A \cup B) \cap (A \cup C)$ [2]

3. Use a Venn Diagram to represent the following expressions:
   a. $A' \cap B'$ [1]
   b. $A \cap B \cap C'$ [2]
   c. $A \cup B$ [1]

4. The universal set A consists of all integers from 11 to 30 (both numbers inclusive). Let B be the set of all odd numbers in A, and C be the set of integers in A which can be wholly divided by 3 (i.e. 12, 15, 18, ..., 30).

   Find
   a. $B \cup C$ [3]
   b. $B' \cap C'$ [3]

5. Given the following Universal set U and its two subsets P and Q, where

   $U = \{ x: x \text{ is an integer}, 0 <= x <= 10 \}$

   $P = \{ x: x \text{ is prime number} \}$

   $Q = \{ x: x^2 < 75 \}$
   a. Draw a VENN diagram for the above [8]
   b. List the elements in $P' \cap Q$ [4]

6. Use a Venn Diagram to represent the following:
   a. $A' \cap B' \cap C$ [3]
   b. $(A \cup B) \cap C'$ [3]

7. If the universal set consists of all integers from -2 to 6 inclusive and $P = \{ -2, 0, 3 \}$ whilst $Q = \{1, 3, 5\}$ what are the elements of: [2]
   a. $P' \cap Q'$ [2]
   b. $(P' \cup Q)'$

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8. A computer school has 33 students, each of whom is sitting examinations in at least one of the following: BCP, RMS, CP. 12 students take all three subjects, 14 take RMS and CP, 16 take BCP and RMS, 18 take BCP and CP, 21 take CP. The same number of students take BCP as take RMS.

a. Put all this information into a Venn Diagram, denoting the number of students who take RMS only and BCP only by X and Z respectively. [5]

b. Find the values of X and Z [5]

c. How many students are sitting only one subject? [2]

9. Given U= { integer x: 1<= x <= 40 }

A = { integers between 1 and 20 inclusive }

B ={ x:$x^2$100)

C = {prime numbers }

D = { multiples of 3 }

List the elements (if any) in

a. $A \cap B$ [2]

b. $B \cap C$ [2]

c. $B \cap D$ [2]

d. $(C \cap D)'$ [2]

e. $B \cap C \cap D$ [2]

10. In a class of 63 students, there are 27 students who can speak Spanish, 24 French and 20 German. There are 7 students who can speak both Spanish and French, 5 both French and German, 6 both Spanish and German, and 6 who speak none of the 3 languages. Let n(S), n(G) denote the number of students who can speak Spanish, French and German respectively. Using the formula:

$\cap (S \cup F \cup G) = \cap(S) + \cap(F) + \cap(G) - \cap(S \cap F) - \cap(S \cap G) - \cap (F \cap G) + (S \cap F \cap G)$

a. Find the number of students who can speak all 3 languages [4] 10

b. Draw a Venn Diagram and fill in the correct number of students in the diagram. [8]

c. Find the number of students who can speak only one of the languages [4] 10

d. What is the probability of any one student selected at random being able to speak exactly 2 of the languages. [2]

e. If a studious Spanish only speaker learnt French,what would be the new probability of any one student selected at random being able to speak exactly 2 of the languages ? [2]

11. A survey of 100 people is conducted to determine how many students excel in the following subjects: Maths (denoted by M), Science (S) and Arts (A). Given:

The number of students who excel in all 3 subjects: 8

The number of students who excel in both S and M: 23

The number of students who excel in both M and A: 20

The number of students who excel in both S and A: 28

The number of students who excel in S only: 24

The number of students who excel in M only: x

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The number of students who excel in A only: y

a. Represent the above information in a VENN diagram [8]

b. Given that the number of students who excel in Maths is more by 8 than those who excel in Arts, derive a pair of simultaneous equations in x and y [6]

c. Solve these equations [3]

d. What us the probability of choosing two people at random, both excelling in only one subject? [3]

12. Use a Venn Diagram to represent the following:

a. $A \cup (B \cup C)'$ [4]

b. $(A \cap B') \cup (A \cap C)$ [4]

13. A survey of fifty athletes found the following to be true:

12 athletes had won no medals;

8 athletes had won only bronze medals;

10 athletes has won only silver medals;

7 athletes has won only gold medals;

2 athletes had won medals of all three types;

8 athletes has won bronze medals and only one other type of medal;

17 athletes had won gold medals
(but some of these had won other medals too)

a. How many athletes had won both bronze and silver medals but not gold medals ? [1]

b. How many athletes had won silver medals and at least one other type of medal ? [1]

c. How many athletes had not won any bronze medals ? [1]

14. A survey of animals at a zoo records the following facts about each nature; whether it has stripes whether it eats meat; whether it climbs trees. Draw separate Venn diagrams for each of the following animals, shading those areas which represent true facts:

a. Zebra (strips, not a meat-eater, doesn't climb trees) [2]

b. Leopard (not strips, meat-eater, climbs trees) [2]

c. Chipmunk (strips, not a meat-eater, climbs tress) [2]

d. Elephant (not strips, not a meat-eater, doesn't climb trees) [2]

15. State whether the following statement are TRUE or FALSE:

a. For sets, A intersect B is equal to B intersect A. [1]

b. For sets, A union B is equal to B union A. [1]

c. For sets, if A is the complement of B, A intersect B is empty [1]

d. For sets, if B is the complement of A, A intersect B is empty [1]

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Probability

Introduction

Probability theory is the science of evaluating uncertainty. It provides the means by which a certain amount of order and predictability may be derived from the natural state of uncertainty.

We can forecast the likelihood of an occurrence by applying probability model on statistical data collected.

6.9 Definition of Probability

We can measure probability by considering how often an event occurs in relation to how often it could occur. It is normally given in terms of a ratio. An event is simply an occurrence.

$$P(E) = \frac{\text{No. of outcomes that satisfy event E}}{\text{No. of total possible outcomes}}$$

Example: Tossing a coin, throwing a dice.

- If a coin, when tossed, has one chance in two of turning up a head, we say that the probability of getting a head is 1/2. When many coins are tossed, it is likely that about one half of them will turn up heads. In symbols, we write this as:

$$P(\text{Head}) = {}^1/_2$$

- If a dice, when tossed, has one chance in six of turning up with the face containing six dots, we say that the probability for the face with six dots to turn up is 1/6. In symbols, we write this as:

$$P(6) = {}^1/_6$$

- In arranging three letters A, D, P in a row, what is the probability that they will appear as PAD. What is the probability that they will appear either as PAD or DAP?

  The letter A, D, P can be arranged as follows :

  PAD, PDA, DAP, DPA, ADP, APD

  Thus, the letters may be arranged in six different ways.
  Only one of them is PAD. Hence, the probability of the letters appearing as PAD is $^1/_6$. In symbols, we write this as:

$$P(\text{PAD}) = {}^1/_6$$

  The arrangement PAD and DAP are two of the total six arrangements. Thus, the probability of the letters appearing either as PAD or DAP is $^2/_6$ i.e. $^1/_3$. In symbols, we write this as :

$$P(\text{PAD or DAP}) = {}^1/_3$$

Note: Probability can never be less than 0 or greater than 1.

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6.10 Success and Failure

In probability theory, if an event occurs, we say we have a success. The probability of a success is defined as :

$$p = \frac{\text{No. of successes}}{\text{No. of possible occurences}}$$

If the event we are concerned with does not occur, we have therefore a failure. The probability of a failure is symbolized as q.

$$q = \frac{\text{No. of failures}}{\text{No. of possible occurences}}$$

Since we have either failure or success for an event, and the total probability is 1, therefore

$$p + q = 1$$

Example:

1. What is the probability of turning up a number greater than 2 with one throw of a dice?

2. What is the probability of turning up a number greater than 0?

Solution:

1. The number greater than 2 which are on the faces of the dice are 3, 4, 5 and 6. Thus 4 successful events are possible i.e. s = 4.

   There are 2 possible unsuccessful events, therefore f = 2
   (i.e.. turning up the number 1 and 2).

$$\text{Hence P (number greater than 2)} = \frac{s}{f+s} = \frac{4}{2+4} = \frac{2}{3}$$

2. The numbers 1, 2, 3, 4, 5 and 6 are all greater than 0. Hence there are no unsuccessful events i.e.. f = 0

$$\text{Therefore P(number greater 0)} = \frac{s}{f+s} = \frac{6}{6} = 1$$

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Example:

What is the probability of drawing a clubs from a pack of 52 cards?
(There are 13 hearts, 13 clubs, 13 diamonds and 13 spades in a pack.

Solution:

$$s = 13$$
$$f = 13 + 13 + 13$$
$$n = f + s = 13 + 13 + 13 + 13 = 52$$

$$\text{Therefore } P(\text{a club}) = \frac{13}{52}$$
$$= \frac{1}{4}$$

where s represents the number of ways a successful event can occur
f represents the number of ways an unsuccessful event can occur
n represents the number of all possible ways an event can occur

The following list gives example of some common probability:

Events	Probability
Certainty of death	$p = 1$
Sun rising tomorrow	$p = 1$
The card you pick is not a heart	$p = \frac{3}{4}$
Getting heads in tossing a coin	$p = \frac{1}{2}$
Newly born baby being a boy (girl)	$p = \frac{1}{2}$
A man living 300 years	$p = 0$

6.11 Probability Spaces

Use Venn diagram to depict all possible outcomes in a problem. The number of elements in each set or subset are clearly visible, when the number of elements of the Universal set is known, probability of an event may be obtained.

Example:

Of 21 typists in an office 5 use (M)annual typewriters, (E)lectronic typewriters AND (W)ord Processors; 9 use (E) and (W); 7 use (M) and (W); 6 use (M) and (E); but no one uses (M) only.

1. Using M, E and W to denote appropriate sets, represent the above information in the form of a Venn diagram.

2. The same number of typists use (E) as use (W). How many use (W) only?

3. How many typists use (E)?

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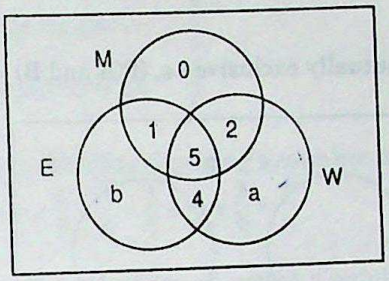
If typists are selected at random and window replacement:

4. What is the probability that any 1 person selected is able to use only 1 type of machine?

5. What is the probability that any 2 people selected can both use word processors?

Solution:

1.



2.

n(E)	=	N(W)	Let a be n(W only)
b + 1 + 5 + 4	=	a + 2 + 5 + 4	Let b be n(E only)
b - a	=	1	①
n(U)	=	21	
n(U)	=	a + b + 1 + 2 + 5 + 4	
21	=	a + b + 1 + 2 + 5 + 4	
a + b	=	9	

í

Solve eqn 1 and eqn 2 for a and b

eqn ① + eqn ②

2b	=	10
b	=	5

Substitute into eqn 1

	5 - a	=	1
⇒	a	=	4

The number of typists who use (W) only is 4

3. n(E) = b + 4 + 5 =1
15 = 15

4. P(any 1 person selected is able to use only one type of machine)

$$= \frac{\text{No. of typist who use only one type of machine}}{\text{No. of typist}}$$

$$= \frac{9}{21}$$

$$= \frac{3}{7}$$

5. P(2 persons selected can both use word processors)

= P(First person can use word processor and second person can use wordprocessor)

$$= \frac{15}{21} * \frac{14}{20}$$

$$= 0.5$$

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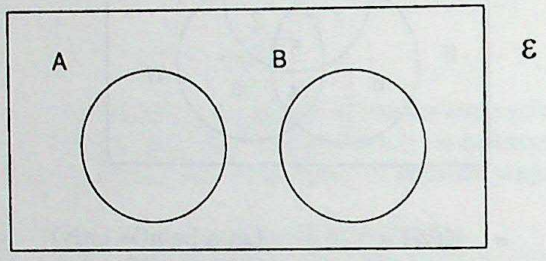
6.12 Probability of Combined Events

6.12.1 Addition Law

If the probability of n mutually exclusive events happening are P_1, P_2, P_3... P_n then the probability that one of these events will occur is :

$P_1 + P_2 + P_3 + ... + P_n$

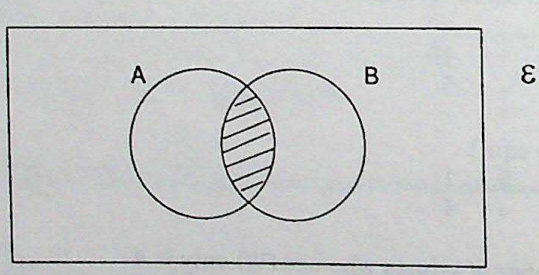
Events A and B are mutually exclusive i.e. (P(A and B) = 0)



6.12.2 General Addition Law

If A and B are two events then

P(A or B) = P(A) +P(B) - P(A and B)



If P(A and B) = 0

→ A and B are mutually exclusive events.

Example: What is the probability of getting a 2 or a 3 with one throw of an ordinary dice?

Solution: Probability of getting a 2 = $\frac{1}{6}$

Probability of getting a 3 = $\frac{1}{6}$

Therefore probability of getting a 2 or a 3 = $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

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Example: What is the probability of getting a number not more than 5 with one throw of a dice?

Solution: Probability of getting a 1 = $\frac{1}{6}$

Probability of getting a 2 = $\frac{1}{6}$

Probability of getting a 3 = $\frac{1}{6}$

Probability of getting a 4 = $\frac{1}{6}$

Probability of getting a 5 = $\frac{1}{6}$

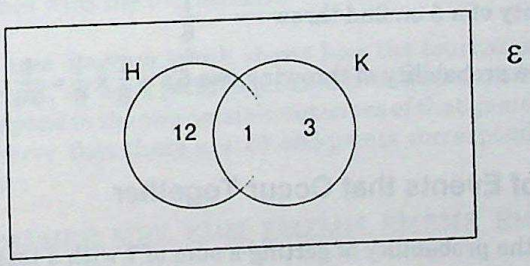
Therefore the probability of getting a number not more than 5 with one throw = $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{5}{6}$

Alternatively, the probability of getting a number more than 5 (i.e. a 6) is $\frac{1}{6}$

Therefore the probability of getting a number not more than 5 $= 1 - \frac{1}{6} = \frac{5}{6}$

Example : A card is drawn from a shuffled pack of 52 cards. What is the probability of drawing a king or a heart?

Solution:



P(king or heart)

= P(king) + P(heart) - P(king and heart)

= $\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{4}{13}$

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6.12.3 Multiplication Law

If the probability of n independent events are $P_1, P_2, P_3, \ldots P_n$ then the probability that ALL the events will occur is :

$$P_1 \times P_2 \times P_3 \times \ldots \times P_n$$

Example: What is the probability of getting four heads in four tosses of a coin?

Solution: Probability of head on 1st throw $= \frac{1}{2}$

Probability of head on 2nd throw $= \frac{1}{2}$

Probability of head on 3rd throw $= \frac{1}{2}$

Probability of head on 4th throw $= \frac{1}{2}$

Therefore probability of getting four heads in four tosses $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

In general, P (A and B) = P(A) * P(B\A)
where P(B\A) means probability of B given that A has occurred.

If A and B are independent then P(B\A) = P(B) (regardless of A)
i.e. P(A and B) = P(A) x P(B)

Example: What is the probability of obtaining 5's with two throws of a dice?

Solution: Probability of a 5 on 1st throw $= \frac{1}{6}$

Probability of a 5 on 2nd throw $= \frac{1}{6}$

Therefore probability of throwing two 5's $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

6.12.4 Probability of Events that Occur Together

Example: What is the probability of getting a sum of 7 with a toss of two dice?

Solution: Sample space:

(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)
(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)

There are altogether 36 sample points.

The set of sample points which give a total of 7 is :

$$E = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$

Therefore the required probability $= \frac{n(E)}{n(S)} = \frac{6}{36}$ or $\frac{1}{6}$

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Example: Suppose we toss a coin and then a dice. What is the sample space for this experiment? What is the probability of tossing a (H, 6)?

Solution: If S represent the sample then

S = {(H,1), (H,2), (H,3), (H,4), (H,5), (H,6),(T,1), (T,2), (T,3), (T,4), (T,5), (T,6)}

Therefore n(S) = 12

There is only one sample point with (H,6) in the sample space.

Therefore n(E) = 1

Therefore $P(E) = \frac{n(E)}{n(S)} = \frac{1}{12}$

where S (sample space) represents all possible events,
E represents the required event.

6.13 Tree Diagrams

A (rooted) tree diagram augments the fundamental principle of counting by exhibiting all possible outcomes of a sequence of events where each event can occur in a finite number of ways.

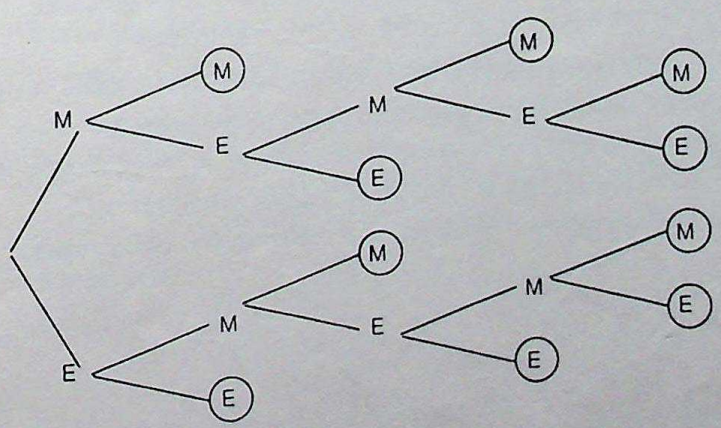
Example:

Marc and Erik are to play a tennis tournaments. The first person to win two games in a row or to win a total of three games wins the tournament.

The figure below gives a tree diagram which shows how the tournament can be. The tree is constructed from left to right. At each point (a game) other than an endpoint, there originate two branches which correspond to the two possible outcomes of that game, i.e. Marx (M) winning or Erik (E) winning. Observe that there are 10 end-points corresponding to the 10 possible courses of the tournament :

MM, MEMM, MEMEM, MEMEE, MEE, EMM, EMEMM, EMEME, EMEE and EE

The path from the beginning of the tree to a particular endpoint describes who won which game in that particular course.



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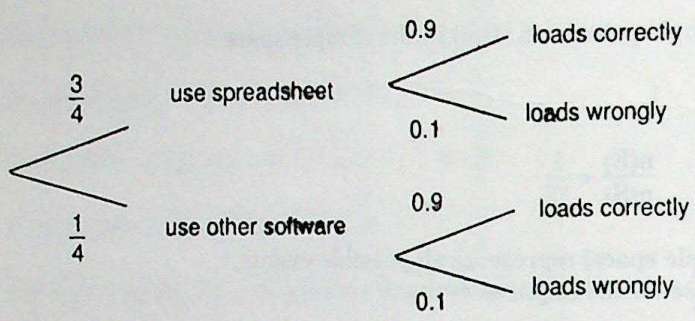
Example:

If the probability that you will use a particular spreadsheet system is 3/4 and the probability that the software loads correctly is 0.9, there are 4 possible results when you try to use that spreadsheet. Draw the appropriate *probability tree* to illustrate this situation and calculate the probability of each of the 4 possible outcomes.

Solution:



P(use spreadsheet, load correctly) = $\frac{3}{4}$ x 0.9 = 0.675

P(use spreadsheet, load wrongly) = $\frac{3}{4}$ x 0.1 = 0.075

P(use others, load correctly) = $\frac{1}{4}$ x 0.9 = 0.225

P(use others, load correctly) = $\frac{1}{4}$ x 0.1 = 0.025

Example: Produce a probability tree to show the eight outcomes of the experiment including the three events that:

The company database needs updating with a probability of $\frac{1}{4}$

The database manager is in a meeting with a probability of $\frac{1}{5}$

The database maintainer is on holiday with a probability of $\frac{1}{8}$

(Hint: Check that the probabilities of these eight outcomes as shown on the tree do indeed total 1).

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Points to Remember

- Definition of probability

  Probability of an event E

  $$P(E) = \frac{\text{No. of outcomes that satisfy event E}}{\text{No. of total possible outcomes}}$$

- Range of Probability

  $0 \leq P(E) \leq 1$

  When P(E) = 0, it means that event E is *not* possible
  When P(E) = 1, it means that event E is certain.

- The occurrence of an event can be treated as success, otherwise a failure.

  - P(Success) + P(Failure) = 1

- Venn diagram from set theory, histogram or cumulative frequency diagram from a statistical frequency distribution; normal distribution, they can form probability spaces on which probability of relevant events can be evaluated.

- Two cases of combined events

  - Probability of one of the mutually exclusive events will occur
    $= P_1 + P_2 + P_3 + ... + P_n$
  - Probability of ALL of the independent events will occur
    $= P_1 * P_2 * P_3 * ... * P_n$

- Tree diagrams are useful to show all possible outcomes of a sequence of events, that helps to determine the probability of a combination of events.

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Past Years Questions (Probability)

1. A computer generates random odd integers from 1 to 9 inclusive. If three numbers are generated, calculate the probability that:
   a. All three numbers are three [2]
   b. At least two of the numbers are five [2]

2. From a production run of 10,000 RAM chips, 4% of which are defective, one chip is selected at random
   a. What is the probability that the chip is defective ? [1]
   b. What is the probability that it is not defective ? [2]

3. There are ten numbers, from 0 to 9, in a bag. Three draws are made at random without replacement from it.
   a. What is the probability of having three different numbers ? [2]
   b. What is the probability that the sum of the three numbers is less than 4 ? [2]

4. A prime number is an integer, greater than or equal to 2, which can only be wholly divided by and by itself. A box contains numbers from 2 to 10 (both numbers inclusive). If 2 numbers are chosen at random without replacement, what is the probability that:
   a. Both are prime numbers ? [2]
   b. Neither of them is prime number ? [2]

5. The probability that a character transmitted is incorrect is 1/8.
   a. Find the probability that three characters transmitted in a row are all correct. [2]
   b. Find the probability that at least one of the three characters is not transmitted correctly. [2]

6. A computer generates random integers from 1 to 6 inclusive. If two numbers are generated, calculate the probability that:
   a. Neither is a 4 [2]
   b. At least one of them is a 4. [2]

7. The probability that the profits of a company will increase in any one year is 0.7
   a. Draw a probability tree to show all probabilities over two years. [2]
   b. Show by marking with an asterisk (*) the nodes which represent an increase in profit in just ONE of the two years. [2]

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8. a. A box contains three pink and five blue balls. The balls are withdrawn from the box one after another. Assuming that the balls are not replaced:

i. Draw a probability tree to show the possible outcomes of three withdrawals. [6]

ii. What is the probability of withdrawing pink,blue, pink balls ? [2]

iii. Find the probability of withdrawing at least two blues balls in three withdrawals. [2]

iv. Find the probability of withdrawing three pink balls in three withdrawals. [2]

b. Assuming that the ball is replaced before the next withdrawal. Repeat all four parts in

a. above [8]

9. There are six apples and twelve oranges in a box. Three draws are made, at random, without replacement.

a. Draw a probability tree to show all the possible outcomes [8]

b. What is the probability of getting first draw an apple, second draw an orange and the third and apple again [2]

c. What is the probability of getting all oranges from the three draws. [2]

10. A box contains 15 discs, of which 3 are defective. 3 discs are drawn at random consecutively, without replacement.

a. Draw a probability tree to show the possible outcomes of the above 3 draws. Use N to represent a draw of a Non-defective disc and D for a draw of a Defective disc. [8]

b. Calculate the probability of drawing 2 discs which are all defective. [2]

11. In a certain factory, the probability of a disc being defective is 6%

a. Draw a probability tree to show all the possible outcomes for two discs chosen at random. [4]

b. Find the probability that at least one disc from the two is defective. [2]

12. a. A basket contains 8 oranges and 2 pears. The fruits are picked at random from the basket, one after another. Assuming the fruits are not replaced:

i. Draw a probability tree to show the possible outcomes of three picks. [7]

ii. Find the probability of picking an orange, a pear and an orange in that order. [2]

iii. Find the probability of picking at least 2 oranges. [3]

b. Recalculate i., ii., iii. above assuming the fruits are picked with replacement. [8]

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13. Eleven football players hang their boots on hooks on a wall. One day, someone who has come to paint the wall takes off all the boots and throws them randomly on the floor.

a. If a player picks up a boot without looking, what is the probability that it is one of his? [2]

b. If a player picks up two boots without looking, what is the probability that they are both his? [2]

c. If a player picks up two boots from the original pile without looking, what is the probability that one is his and one is someone else ? Give your answer as fractions, rather than as decimals. [2]

14. A student keeps 10 pens in a drawer. Although the pens all look the same, 2 of them have red ink, 3 of them have black ink, and the remainder have blue ink.

a. The student takes two pens from the drawer at random, one after the other. Assuming that the pens are not replaced:

i. Draw a probability tree to show the possible outcomes from two picks. [6]

ii. Find the probability of picking both of the pens that have red ink. [2]

iii. Find the probability of picking at least one pen with black ink. [2]

b. The student puts all the pens back in the drawer, and this time removes the pens, tests them, and replaces them.

i. Draw a probability tree to show the possible outcomes from two picks. [6]

ii. Find the probability of picking a pen with red ink on both occasions. [2]

iii. Find the probability of picking at least one pen with blue ink. [2]

15. A man throws a six-sided dice three times in a row. What is the probability that he:

a. Rolls three sixes [1]

b. Rolls no sixes [1]

c. Rolls the same number each time [1]

d. Rolls different numbers each time [2]

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Chapter 7

Boolean Algebra (I)

Chapter Objectives

At the completion of this chapter, you would have learnt:

- the importance of two-state(TRUE/FALSE) logic in computing;
- model logical relations by truth tables, Venn diagrams, switching circuits or 'gates'.

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7.1 Introduction

In programming, especially in selection or branching statements, we need to specify conditions. These conditions are either satisfied or not satisfied, in other words they are either TRUE or FALSE. Thus understanding of this two-state (TRUE or FALSE) logic is fundamental in programming, it will help to identify many logic errors that would be made otherwise.

In digital electronics, devices like digital computers, calculators fundamentally work on two-state, 1 or 0, (ON or OFF). There are tiny circuit elements which operate on the two states, they are called Logic Gates. In this chapter, we will know more about AND, OR, NOT, NAND.

7.2 Logic in Programming

Sooner or later, in all but the simplest of programs, we meet a situation where we have to write something like "IF so-and-so THEN DO something, ELSE DO something different". This means that we must be able to model the problem in two-state logical terms, i.e. outcome is YES or NO.

In many cases, we may be making several logical decisions within a single line of code. If, for example, we are sorting data into the classes which we used for our statistical distribution of the run-times, we might well do so by a line such as:

```
IF x < 7 THEN
      C1 ← C1 + 1
ELSE IF X < 11 THEN
               C2 ← C2 + 1
      ELSE  IF X < 15 THEN
                  C3 ← C3 + 1
               ELSE  C4 ← C4 + 1
               ENDIF
      ENDIF
ENDIF
```

This involves *compound* decisions within which each single decision must be logically correct, without any doubt whatever.

This is the realm of two-state logic, also referred to as Binary Logic and we use '1' for *true* or *yes* and '0' for *false* or *no*.

7.3 Logic and Hardware

We shall also need to know precisely how the logical two-state system maps on to a physical two-state switching system if we become involved with assembly languages or the theory and architecture of the computer, or if we need to program any process control applications.

Today such systems, from controlling traffic lights to running the cycles of a washing machine, embody the logical models in various arrangements of miniaturized transistor switches in 'chips'. In this context, '*yes*' ordinarily translates as 'switch on' and '*no*' as 'switch off'.

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7.4 Language and Symbols

A different language & some new symbols :

- STATEMENT
  - Used in two-state logic to mean any single proposition which can only be TRUE or FALSE.

We now venture into total certainty, which associates YES with TRUE and binary 1; NO with FALSE and binary 0.

Statements may be COMPOUND, when we associate two or more by the conjunction AND, or by the disjunction OR.

We often use the small letters p, q and r as the symbols for this sort of logical statement. In particular, the compound statement p ^ q means 'the consequences' of the two statements in conjunction. The consequences refer to the four possible results of any compound of two statements.

Similarly, p v q indicates the possible results of the disjunction of p, q. 'NOT p' is symbolized by ~p.

7.5 Truth Tables

Let us now use our primary tool for simplifying the NOT, AND and OR models, the truth table.

p	q	p^q (p AND q)	p v q (p OR q)
False	False	False	False
False	True	False	True
True	False	False	True
True	True	True	True

In logic, we distinguish between OR. meaning in either of the named sets or their intersection, and XOR (exclusive OR), which means in either but not both.

7.6 Logical Equivalence

The use of truth tables leads also to another condition, the *logical equivalence* of compound statements. If the outcomes of two truth tables are identical, the statements which they represent are logically equivalent and we use the '=' symbol to represent this state.

For example, ~(p ^ q) = (p~ v ~q)

p	q	p^q	-(p^q)	-p	-q	-p v -q
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

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The compound statements will often include more than two propositions and the table may then become quite large.

If the compound statement were 'p AND q OR r' we might find that the interpreter we are using automatically takes the 'AND' before the 'OR', but is often safer to use brackets whilst we are working on the model and write it as (p^q) v r and to insert result, which is p ^ q:

p	q	(p^q)	r	(p^q) v r
0	0	0	0	0
0	0	0	1	1
0	1	0	0	0
0	1	0	1	1
1	0	0	0	0
1	0	0	1	1
1	1	1	0	1
1	1	1	1	1

With three statements, we must ensure that the program include all the combinations which can give rise to 8 possible results.

Example: For the boolean expression

$Z = A + B \cdot \overline{C}$

or

Z = A OR (B AND (NOT) C)

The truth table is as follows:

A	B	C	$\overline{C}$	$B \cdot \overline{C}$	$Z=A+B \cdot \overline{C}$
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	1	1	1
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	0	0	1
1	1	0	1	1	1
1	1	1	0	0	1

Example: For the boolean expression

$Z = (A \cdot B) + \overline{C} + D$

or

Z = (A AND B) OR (NOT C) OR D

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The truth table is as follows:

A	B	C	D	A . B	$\overline{C}$	Z=(A . B)+$\overline{C}$+D
0	0	0	0	0	1	1
0	0	0	1	0	1	1
0	0	1	0	0	0	0
0	0	1	1	0	0	1
0	1	0	0	0	1	1
0	1	0	1	0	1	1
0	1	1	0	0	0	0
0	1	1	1	0	0	1
1	0	0	0	0	1	1
1	0	0	1	0	1	1
1	0	1	0	0	0	0
1	0	1	1	0	0	1
1	1	0	0	1	1	1
1	1	0	1	1	1	1
1	1	1	0	1	0	1
1	1	1	1	1	0	1

Truth Tables can be used to prove the logic equality of two different boolean expressions, as shown in the example below.

Example: To prove:
A . (A + B) = A + A . B = A

Truth table for A . (A +B)

A	B	A+B	A . (A + B)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

Truth table for A + A . B

A	B	A . B	A + A . B
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

The two expressions A . (A + B) and A + A . B are shown to be logically equivalent, because they have the same outputs in the above truth tables.

Note also that these outputs are also exactly the same as the inputs for A.

Therefore it is proven that
A . (A + B) = A + A . B = A

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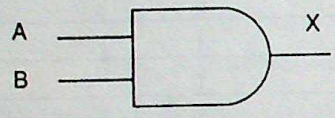
7.7 Switching Diagrams

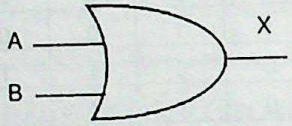
It has long been the practice to illustrate such possibilities, in *Control Systems*, by reference to diagrams showing the switching *On* and *Off* of an electric light.

The switching function is carried out, within the computer, by circuits known as *Logic Gates*, for which statements have to be converted to electrical voltages, still identified as '1' or '0' for *'On'* and *'off'* but with individual statements, or *Inputs*, labelled as A, B, C, and so on whilst consequences, or *Outputs*, are labelled X, Y, Z.

These 'gates' are represented in diagrams as shown below but, just as with a truth table, compound statements may give any number of 'inputs' and gates may be combined in many different ways.



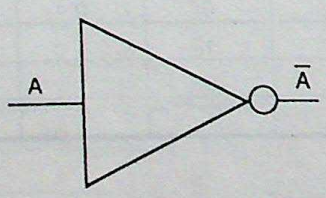
The AND gate



The OR gate

Here A ^ B is represented as A.B and A v B is represented as A + B. So the two-input AND gate means that to generate a voltage, indicated by '1' at the output, we must apply voltage at all inputs. For the OR gate, a voltage at the output is generated by an appropriate voltage at any one or more inputs.

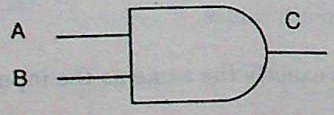
If we have a statement, 'IF in A AND NOT in B THEN DO ...', then we see that we need to invert an input (or an output) to conform to our logical model.



The NOT gate

7.7.1 AND Function (or AND gate)

The AND gate circuit symbol is as follows:



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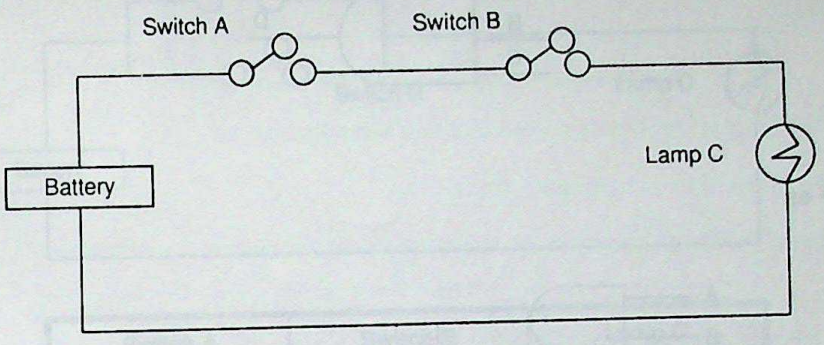
A basic AND gate has two inputs A and B, and one output C. All A, B and C are logical and binary variables which can only be '0' or '1' i.e. A, B and C are bits in nature.

The analogy of the AND gate is as follows:



Switch A	Switch B	Lamp C
OFF	OFF	OFF
OFF	ON	OFF
ON	OFF	OFF
ON	ON	ON

The AND function output C is a 1 (True) *only* if both inputs A = 1 AND B = 1, or else, output C = 0. This can be better represented by a truth table which relates the output to all possible input conditions.

A Truth Table shows the relationship between input conditions and output.

'AND'

Switch A	Switch B	Lamp C
0	0	0 'Low'
0	1	0 'Low'
1	0	0 'High'
1	1	1 'High'

When input A and B are both '1', the output will be '1'.

Mathematically, this can be expressed in a boolean equation as:

$C = A \cdot B$ (*read as A AND B*)

or

$C = A \cap B$

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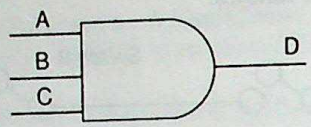
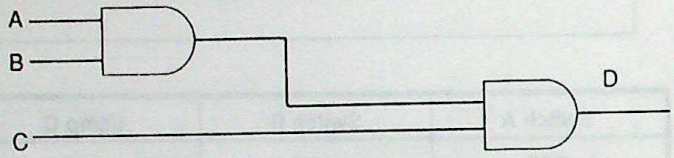
Example: A 3-input AND gate:



is the same as

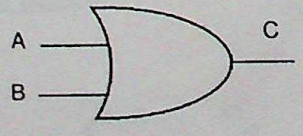


The truth table is as follows:

Input			Output
A	B	C	D
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

7.7.2 OR Function (or OR gate)

The OR gate circuit symbol is as follows:



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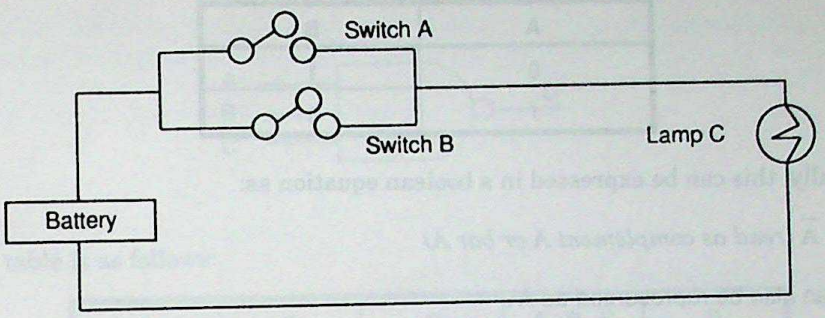
The analogy of an OR gate is as follows:



Switch A	Switch B	Lamp C
OFF	OFF	OFF
OFF	ON	ON
ON	OFF	ON
ON	ON	ON

It can also be represented by the following truth table:

Switch A	Switch B	Lamp C
0	0	0
0	1	1
1	0	1
1	1	1

That is, an OR gate output C = 1 if either A = 1 OR B = 1. Otherwise C = 0

Mathematically, this can be expressed in a boolean equation as :

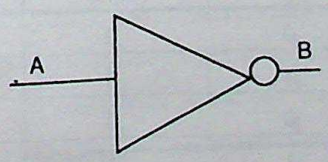
$$C = A + B \text{ (read as A OR B)}$$

or

$$C = A \cup B$$

7.7.3 NOT Function (or NOT gate)

The NOT gate circuit symbol is as follows:



The action of a NOT gate is quite simple. It functions like an inverter. That is, the output is the inverted value of the input. For example, if A = 0 then B = 1, and if A = 1 then B = 0.

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The truth table is as follows:

A	B
0	1
1	0

Mathematically, this can be expressed in a boolean equation as:

$B = \overline{A}$ *(read as complement A or bar A)*

Note: $\overline{A}$ can also be represented as A'.

7.7.4 Other Logical Functions

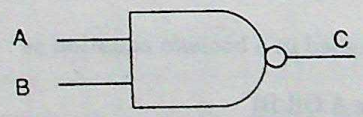
AND, OR and NOT are known as the basic gates. There are other gates and logical functions used in computers which are built from the three basic gates.

- NAND function (i.e. NOT-AND function)
- NOR function (i.e. NOT-OR function)

NAND and NOR gates are known as the universal building elements which are used to build more complex logical functions like the Adder *(which we will cover later in the chapter).*

NAND gate

The NAND gate circuit symbol is as follows:



Mathematically, this can be expressed in a boolean equation as:

$C = \overline{A \cdot B}$

The truth table is as follows:

A	B	A . B	C
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

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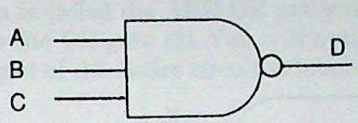
The 3-input NAND gate circuit symbol is as follows:



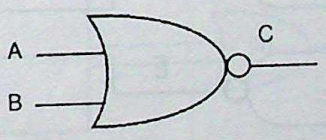
The truth table is as follows:

A	B	C	A.B.C	D
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0

NOR gate

The NOR gate circuit symbol is as follows:

Mathematically, this can be expressed in a boolean equation as:



$$C = \overline{A+B}$$

The truth table is as follows:

A	B	A + B	C
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

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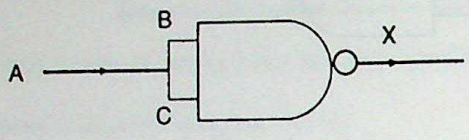
NAND & NOR gate used as an inverter

NAND gate (Universal gate)

NAND gate can be modified to function as NOT gate, OR gate.



Note: B, C inputs follow from A.

NOT function

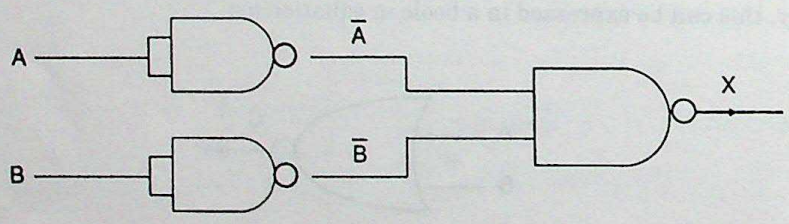
A	B	C	X
0	0	0	1
1	1	1	0

OR function

$x = A + B$

$= \overline{\overline{A + B}}$ (Double negation)

$= \overline{\overline{A} \cdot \overline{B}}$ (DeMorgan's Law)



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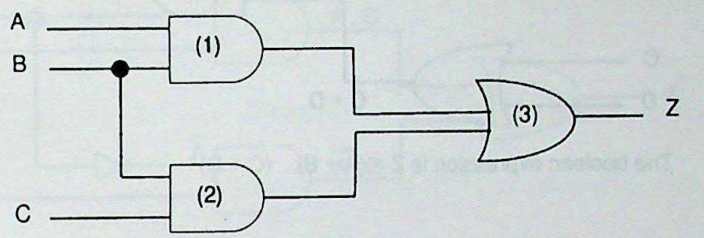
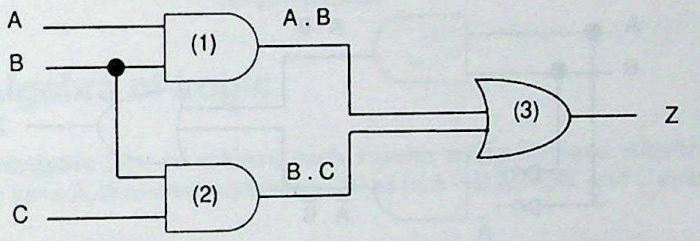
7.8 Combining Logic Gates

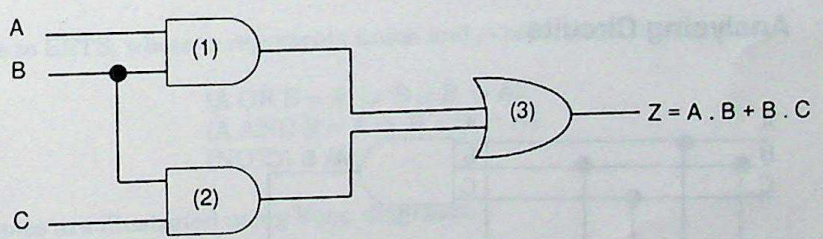
Many everyday digital logic problems use several logic gates. The most common pattern of gates is shown below. This pattern is called the AND-OR pattern. The outputs of the AND gates (1 and 2) are used as inputs to the OR gate (3). You will notice that this logic circuit has three inputs (A, B and C). The output of the entire circuit is labeled as Z.



(a) AND-OR logic circuit



(b) Same circuit with boolean expressions at the outputs of the AND gates.



(c) Same circuit with boolean expressions at the outputs of the OR gates.

Let us first determine the boolean expression that will describe this logic circuit. Begin the examination at gate (1). This is a 2-input AND gate. The output of this gate is A . B (A AND B). This expression is written at the output of gate(1) in figure (b) above.

Gate(2) is also a 2-input AND gate. The output of this gate is B . C (B AND C). This expression is written at the output of gate(2). Next the output of gate(1) and (2) are OR-ed together by gate(3). Figure (c) shows A . B being OR-ed with B . C.

The resulting boolean expression is Z = A . B + B . C. It is read as (A AND B) OR (B AND C). You will notice that the AND-ing is done first, followed by the OR-ing.

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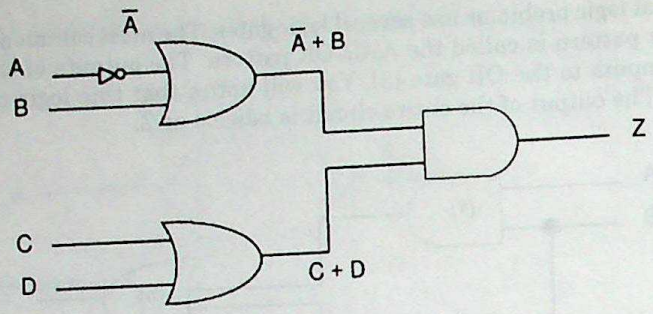
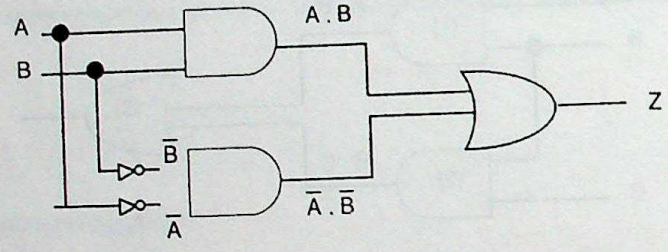
Example: A logic circuit is given as follows:



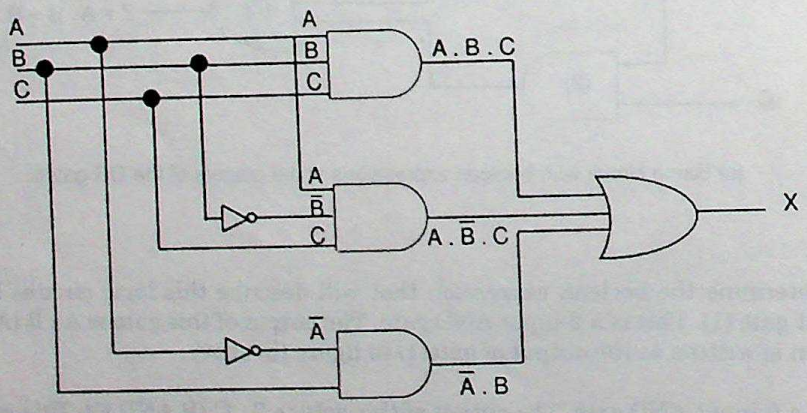
The boolean expression is $Z = (\bar{A} + B) . (C + D)$

Example: A logic circuit is given as follows:



The boolean expression is $Z = A . B + \bar{A} . \bar{B}$

7.8.1 Analysing Circuits



$X = A . B . C + A . \bar{B} . C + \bar{A} . B$

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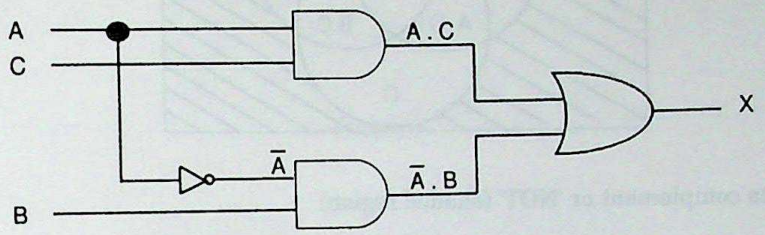
From the diagram, we see that each of the outputs from the 3 AND gates will be a '1' or a '0' and they serve as the inputs to the 'OR' gate which is represented by the '+' sign.

Switching circuits are often over-complicated because they duplicate or even triplicate or quadruplicate functions. The results of rigorous simplification can, however, be quite startling.



$X = \overline{A} . B + A . C$
Simplified circuit

7.9 The Algebra of Logic

We shall now investigate how to achieve such results using Boolean Algebra. Note that in two-state logic, we have A, $\overline{A}$ and two operators, '+' as in A + B for 'OR' and '.' as in A.B for 'AND'.

Using these symbols, we have

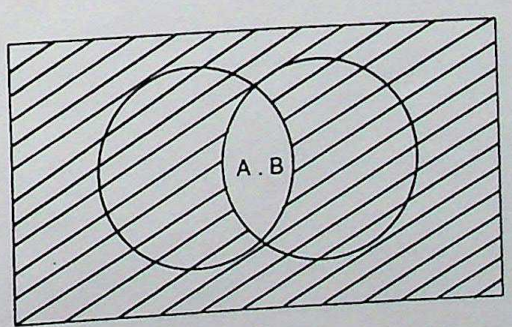
$A + \overline{A} = 1$	$A.\overline{A} = 0$
$A + 0 = A$	$A . 1 = A$
$A.0 = 0$	$A + 1 = 1$

With reference to SETS, where $\cup$ represents union and $\cap$ represents intersection.

$A + B = B + A$	(A OR B = $A \cup B = B \cup A$)
$A.B = B.A$	(A AND B = $A \cap B = B \cap A$)
	(NOT A = A')

Further examples are illustrated using Venn diagrams:

$\overline{A . B}$



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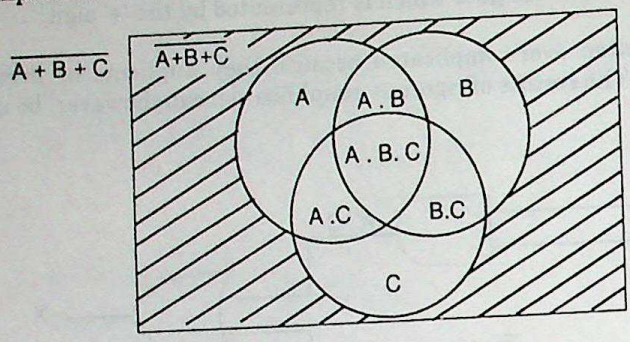
A.B and its complement of 'NOT' (shaded region)



A + B + C and its complement or 'NOT' (shaded region)

Points to Remember

Logic Operators	Logic Gates		Set Notation
A AND B	$A \wedge B$	A . B	$A \cap B$
A OR B	$A \vee B$	$\underline{A}$+B	$A \cup B$
NOT A	~ A	$\underline{A}$	A'

- AND operator returns *true* if all inputs are *true or* operator returns *true* if one input is *true.*
- Truth Table is used to show all possible *outputs* with all possible combination of *inputs.*

2 Inputs give rise to $4(=2^2)$ combinations
3 Inputs give rise to $8(=2^3)$ combinations
4 Inputs give rise to $16(=2^4)$ combinations

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Past Years Questions

Refer to Chapter 8

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Chapter 8

Boolean Algebra (II)

Chapter Objectives

At the completion of this chapter, you would have learnt:

- to analyse and simplify circuits by Boolean Algebra;
- to draw a logic circuit.

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8.1 Introduction

In addition to our 'ordinary' Boolean algebra, De Morgan's Laws is one tool used for simplifying complex expressions in Boolean algebra or complex switching circuits.

8.2 The Laws of Boolean Algebra

Commutative law

$$A \cdot B = B \cdot A$$
$$A + B = B + A$$

Associative law

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C = A \cdot B \cdot C$$
$$A + (B + C) = (A + B) + C = A + B + C$$
$$(A + B) + (C + D) = A + B + C + D$$

Distributive law

$$A \cdot (B + C) = A \cdot B + A \cdot C$$
$$(A + B)(A + C) = A \cdot A + A \cdot B + A \cdot C + B \cdot C = A + B \cdot C$$

Boolean rules

$$A \cdot 0 = 0$$
$$A \cdot \overline{A} = A$$
$$A \cdot A = 0$$
$$A \cdot 1 = A$$
$$A + 1 = 1$$
$$A + \overline{A} = A$$
$$A + A = 1$$
$$A + 0 = A$$

Properties of absorption

$$A + (A \cdot B) = A$$
$$A \cdot (A + B) = A$$
$$A + (\overline{A} \cdot B) = A + B$$
$$A \cdot (\overline{A} + B) = A \cdot B$$

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8.3 Simplification of Expressions

The rules given may be used to simplify Boolean expression, just as the rules of normal algebra may be used to simplify expression. Consider the expression

$$(A + B)(A + \overline{B})(\overline{A} + C)$$

The first two terms consist of $A + B$ and $A + \overline{B}$; these terms may be multiplied and, since $A + A\overline{B} + AB = A$ and $B\overline{B} = 0$, reduced A.

The expression has been reduced now to $A(\overline{A} + C)$, which may be expressed as $A\overline{A} + AC$. And since $A\overline{A}$ is equal to 0, the entire expression $(A + B)(A + \overline{B})(\overline{A} + C)$ may be reduced to AC.

Another expression that may be simplified is $ABC + A\overline{B}C + AB\overline{C}$. First, the expression may be written $A(BC + \overline{B}C + B\overline{C})$. Then again, we may simplify to $A[B(C + \overline{C}) + \overline{B}C]$; and since $C + \overline{C} = 1$, we have $A(B + \overline{B}C)$.

The expression $A(B + \overline{B}C)$ may be further reduced to $A(B + C)$. The final expression can be written in two ways: $A(B + C)$ or $AB + AC$. The first expression is generally preferable if the equation is to be constructed as an electronic circuit, because it requires only one AND circuit and one OR circuit.

8.4 De Morgan's Laws

The following two rules constitute De Morgan's Laws:

$$\overline{(A + B)} = \overline{A}.\overline{B}$$

$$\overline{(A \,.\, B)} = \overline{A} + \overline{B}$$

The complement of any Boolean expression, or part of any expression, may be found by means by this laws. Using these rules, two steps are used to form a complement:

Step 1 The + symbols are replaced with . symbols and . symbols with + symbols.

Step 2 Each of the terms in the expression is complemented.

The use of De Morgan's laws may be demonstrated by finding the complement of the expression $A + BC$. First, note that a multiplication sign has been omitted, and the expression could be written $A + (B.C)$. To complement this, the addition symbol is replaced with a multiplication symbols, and the two terms are complemented, giving $\overline{A}.(\overline{B}.\overline{C})$; *then the remaining term is complemented :* $\overline{A}(\overline{B} + \overline{C})$. *The following equivalence has been found :* $(A + BC) = \overline{A}(\overline{B} + \overline{C})$.

The complement of $\overline{A}B + C\overline{D}$ may be formed by two steps:

Step 1 The addition symbol is changed.

Step 2 The complement of each term is formed : $\overline{(\overline{A}.B)\,(C\overline{D})}$

This become $(A + \overline{B})(\overline{C} + D)$

Since A and D were already complemented, they become uncomplemented by the laws $\overline{\overline{A}} = A$.

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An example: Show that $\overline{A}.\overline{B} + \overline{A.B} = \overline{A.B}$

L.H.S = $\overline{A}.\overline{B} + (\overline{A} + \overline{B})$

= $\overline{A}.\overline{B} + \overline{A}.1 + \overline{B}$

= $\overline{A}.\overline{B} + \overline{A}.(B + \overline{B}) + \overline{B}$

= $\overline{A}.\overline{B} + \overline{A}.B + \overline{B}$

= $\overline{A}(\overline{B} + B) + \overline{B}$

= $\overline{A} + \overline{B}$

= $\overline{A.B}$ = R.H.S

The problem may also be solved by drawing a truth table:

A	B	$\overline{A}$	$\overline{B}$	A.B	$\overline{A.B}$	$\overline{A}.\overline{B}$	$\overline{A.B} + \overline{A}.\overline{B}$
0	0	1	1	0	1	1	1
0	1	1	0	0	1	0	1
1	0	0	1	0	1	0	1
1	1	0	0	1	0	0	0

Law of double complementation

$\overline{\overline{A}} = A$

$\overline{\overline{A+B}} = A+B$

$\overline{\overline{A . B}} = A . B$

Example: Simplify $(A + B) . (A+B) + A . (\overline{A} + B)$

Solution: $(A + B) . (A + B) + A . (\overline{A} + B)$

= $A + B + A . (\overline{A} + B)$

= $A + B + 0 + A . B$

= $A + A. B + B =$ $A + B$

Example: Simplify $\overline{\overline{A} + (\overline{B . C}) + \overline{C . B}}$

Solution: $\overline{\overline{A} + \overline{B . C} + \overline{C . B}}$

= $\overline{\overline{A}} . \overline{\overline{B . C}} . \overline{\overline{C . B}}$

= $A . B . C (\overline{\overline{C} + \overline{B}})$

= $A . B . C (C + \overline{\overline{B}})$

= $A . B . C + A . B . C . \overline{\overline{B}}$

= $A . B . C$

Points to Remember

Boolean algebra and De Morgan's laws are two tools to simplify complicated logic expressions or logic circuits.

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Past Years Questions

1. Draw the logic circuit for the following expression using NAND gates only.
$\overline{\overline{A}\,\overline{B}\,\overline{C}}$ [4]

2. Using Boolean algebra, simplify:
$\overline{(A + C).\,\overline{BC} + \overline{AC} + B}$ [5]

3. Using Boolean algebra simplify
$AB\overline{C} + AB\,(\overline{A}B\overline{C} + \overline{(A + \overline{B} + C)}$ [3]

4. a. Using a Boolean algebra, simplify the expression: $A(B + ABC + \overline{A}C)$. [5]
   b. From the simplified expression, draw the logic circuit. [2]

5. a. Use Boolean Algebra to show that:
   $(A.B+B.C+A.C).\overline{(A.B.C)} = \overline{A}.B.C + A.\overline{B}.C + A.B.\overline{C}$ [4]
   b. Draw a logic circuit for the expression
   $\overline{A}B.C + A.\overline{B}.C + A.B.\overline{C}$ [4]

6. a. If $X = \overline{A.B} + \overline{C + \overline{A}}$, create a TRUTH TABLE for X. [5]
   b. Using the truth table, show that X can be represented by a single NAND gate. [2]

7. If $X = \overline{A\overline{B}} + C$
   a. Using a De Morgan's Laws only, prove that
   $X = \overline{A} + B + C$ [4]
   b. Represent $\overline{X} = \overline{A\overline{B}} + C$ using NAND gates only. [4]

8. a. A, B and C are switches in the electrical circuit shown below. If a switch is on, it has a Boolean value of '1', otherwise it has 'O'. The light bulb will light if C = 1 and either A = 1 or B = 1. The light bulb 'X' has a Boolean value of '1' if it is lit, otherwise it has the value 'O'.
      i. Draw a truth table relating the inputs (A, B and C) to the output X. [4]
      ii. Write down, but do not simplify the Boolean. Expression of X. [4]
      iii. Simplify the expression using a Boolean Algebra. [4]
      iv. Redesign the circuit using NAND gates only. [4]
   b. Using only De Morgan's law and Boolean algebra simplify:
   $\overline{(A+C).\overline{AB} + \overline{BC} + \overline{A}}$ [4]

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9. a. Simplify using Boolean algebra $\overline{\overline{AB} + A\overline{B}}$ [4]

b. From the following truth table, obtain the expressions for 'yes' and 'no' for X and simplify them using Boolean algebra [10]

A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

c. Construct the logic circuit for simplified 'yes' and 'no'. [6]

10. In a beauty contest, three judges A, B and C can register their votes as 'I' or 'O' through switches allocated to them. Contestants will be disqualified if two or more judges register 'O' votes for them.

a. Construct a truth table for the above [6]

b. Use a Boolean Algebra to obtain a simplified Boolean expression for the outputs 'qualified' and 'disqualified'. [10]

c. Construct a logic circuit for the simplified disqualified output. [4]

11. a. Find an expression for X from the following truth table and simplify it using a Boolean algebra. [6]

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

b. Draw a logic circuit for the expression derived in part a. above. [4]

12. a. Simplify the following expression using Boolean Algebra:

$\overline{(\overline{AB} + AC).\overline{(A.\overline{B} + \overline{(B + C)})}}$ [8]

b. Using a Boolean algebra only, prove that the expression $A.\overline{(AB + BC)} = A \cdot \overline{B}$ [6]

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c. Obtain an expression for X from the following truth table, and simplify it using a Boolean algebra. [4]

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

d. Draw the logic circuit for the expression derived in part (c) [4]

13. When $X = \overline{(\bar{A} + C).\overline{(A.\bar{B} + C)}}$

a. Create a TRUTH TABLE for X including columns for $(\bar{A}+C)$,$(A.\bar{B}+C)$ and $\overline{(A.B+C)}$ [6]

b. Produce a Boolean expression for this table [2]

c. Use ONLY De Morgan's laws and Boolean algebra to simplify the original expression for X. [4]

14. A manufacturing process has three status checks, A,B and C. The process continues without interruption, if one of the following conditions holds:

all three have a value of 1,
all three have value of 0,
A has a value of 1 and ONE of B or C has a value of 0
A has a value of 0 and C has a value of 1

a. If the process continues, write down, in terms of A, B and C an appropriate Boolean expression. [4]

b. Use a Boolean,algebra to simplify this expression. [6]

c. If the process is interrupted, an electronic signal is produced. Draw the logic circuit necessary to produce such a signal. [5]

15. Using Boolean Algebra prove [5]

$$\overline{A\,(B + AC)} + \overline{AC + \bar{A}\,\bar{B}C + BC} = \bar{A} + \bar{B} + \bar{C}$$

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16. A spaceship is exploring the distant planet IDCS. There are four components of the spaceship which are important for landing: the rockets (R); the navigation system (No); the wheels (W), the hull (H). Because the planet IDCS is so for away, these components break down quite often. The spaceship can only land if one or more of the following conditions are met:

i. the hull (H) is functioning and either the rockets (R) or the wheels (W) are functioning.

ii. the rockets (R) and wheels (W) are both functioning.

iii. the navigation (N) system is functioning and the rockets (R) are functioning.

a. Construct a truth table which indicates whether landing is possible or not, for all values of R, W, N and H. [8]

b. Write a Boolean expression which states the conditions under which the spaceship can land. Simplify your answer as much as possible. [4]

c. Design a logic circuit to implement the Boolean expression you derived as your answer to part (b). Use only 2-input AND, 2-input OR and /or l-input NOT gates (as many of each as is necessary) [4]

d. Suppose the spaceship is approaching to land with all its components in full working order, when the following events occur, in order:

i. the rockets fail
ii. the navigation system fails
iii. the wheels fail iv. the hull fails

After which event will the spaceship be unable to land ? [2]

e. If, instead of having four components important for landing, a new, improved spaceship had six such components, how many entries would be needed in a truth table to cover all the possible combinations of components' functionality? [2]

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Chapter 9

Matrices And Transformations

Chapter Objectives

At the completion of this chapter, you would have learnt:

- to define a matrix and the types of matrix;
- to understand the order of a matrix;
- to perform simple algebra on matrix;
- to relate shapes and their movements in space through matrices;
- to understand the motion and transforms of 2 dimensional shapes;
- model shapes and operate on them by matrix arithmetic;
- to perform a range of elementary 2 dimensional spatial operations.

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9.1 Introduction

Here, we exploit the ability to emulate and to manipulate entities in spatial terms. Ultimately these basic ideas extend to controlling machine and its tools.

Some of us may have seen this in computer applications such as DTP, CAD or CAM.

9.2 Definitions

- A matrix is a rectangle array of m rows and n columns, where m and n are natural numbers. Such a matrix is said to be of order m x n.

Example:

$\begin{pmatrix}1&2\\3&4\\5&6\end{pmatrix}, \begin{pmatrix}a&b&c\\d&e&f\end{pmatrix}$ and $\begin{pmatrix}-9&0\\-1&2\end{pmatrix}$ are matrices

Their orders are 3 x 2, 2 x 3 and 2 x 2 respectively.

An m x 1 matrix is known as a column vector or column matrix.

An n x n matrix is known as a square matrix

A 1 x m matrix is known as a row matrix

- The matrix $\begin{pmatrix}1&0\\0&1\end{pmatrix}$ is called the unit matrix. It is usually represented by the symbol I. When any matrix is multiplied by the unit matrix. The matrix remains unchanged. That is I . A = A . I = A where A is a matrix

Example:

$$\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix} = \begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix} = \begin{pmatrix}a&b\\c&d\end{pmatrix}$$

- An matrix whose elements are all zeros is called a zero matrix.

Example:

$\begin{pmatrix}0&0\\0&0\end{pmatrix}\begin{pmatrix}0&0&0\\0&0&0\end{pmatrix}$ and $\begin{pmatrix}0\\0\end{pmatrix}$ are all zero matrices

When any matrix is multiplied by the zero matrix. The result is a zero matrix.

Example:

$$\begin{pmatrix}0&0\\0&0\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}=\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}0&0\\0&0\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

The zero matrix is sometimes written simply as 0 so that if $A = \begin{pmatrix}a&b\\c&d\end{pmatrix}$ is multiplied by 0, we could write it as : 0 . A = A . 0 = 0

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9.3 Rules/Matrix Algebra

- Two matrices can be added only if they are of the same order. The rule for addition is shown by the following example.

Example:

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & c+f \\ b+g & d+h \end{pmatrix}$$

Example:

$$\begin{pmatrix} 4 & 9 \\ 3 & 5 \end{pmatrix} + \begin{pmatrix} 7 & 8 \\ 1 & 9 \end{pmatrix} = \begin{pmatrix} 4+7 & 9+8 \\ 3+1 & 5+9 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 17 \\ 4 & 14 \end{pmatrix}$$

- Two matrices can be subtracted only if they are of the same order. The rule for subtraction is shown by the following example.

Example:

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} - \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix} + \begin{pmatrix} -e & -f \\ -g & -h \end{pmatrix}$$

$$= \begin{pmatrix} a-e & c-f \\ b-g & d-h \end{pmatrix}$$

Example:

$$\begin{pmatrix} 4 & 9 \\ 3 & 5 \end{pmatrix} - \begin{pmatrix} 7 & 8 \\ 1 & 9 \end{pmatrix} = \begin{pmatrix} 4-7 & 9-8 \\ 3-1 & 5-9 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix}$$

- The rule for multiplying matrices is :

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+cg & af+ch \\ be+dg & bf+dh \end{pmatrix}$$

Multiplying other matrices is done in the same manner but the number of columns of the first matrix must be the same as the number of rows of the second matrix.

Example:

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} ae + cf \\ be + df \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}\begin{pmatrix} g & h \\ i & j \end{pmatrix} = \begin{pmatrix} ag + bi & ah + bj \\ cg + di & ch + dj \\ eg + fi & eh + fj \end{pmatrix}$$

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Example:

Multiply $\begin{pmatrix}4 & 9\\3 & 5\end{pmatrix}\begin{pmatrix}6\\2\end{pmatrix}$

Solution:

$$\begin{pmatrix}4 & 9\\3 & 5\end{pmatrix}\begin{pmatrix}6\\2\end{pmatrix} = \begin{pmatrix}4x6 + 9x2\\3x6 + 5x2\end{pmatrix}$$

$$= \begin{pmatrix}24 + 18\\18 + 10\end{pmatrix}$$

$$= \begin{pmatrix}42\\28\end{pmatrix}$$

Example:

Multiply $\begin{pmatrix}4 & 9\\3 & 5\end{pmatrix}\begin{pmatrix}2 & 4\\1 & 3\end{pmatrix}$

Solution:

$$\begin{pmatrix}4 & 9\\3 & 5\end{pmatrix}\begin{pmatrix}2 & 4\\1 & 3\end{pmatrix} = \begin{pmatrix}4x2 + 9x1 & 4x4 + 9x3\\3x2 + 5x1 & 3x4 + 5x3\end{pmatrix}$$

$$= \begin{pmatrix}8 + 9 & 16 + 27\\6 + 5 & 12 + 15\end{pmatrix}$$

$$= \begin{pmatrix}17 & 43\\11 & 27\end{pmatrix}$$

- If A and B represent two matrices, then

  $AB \neq BA$

  i.e. in matrices, the order of multiplication is not commutative.

- Two matrices are equal if and only of their corresponding elements are equal.

  Thus, if $\begin{pmatrix}a & b\\c & d\end{pmatrix} = \begin{pmatrix}e & f\\g & h\end{pmatrix}$

  then it follows that a = e, b = f, c = g and d = h

  If $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$ then it follows that x = 2 and y = 3

- When a matrix is multiplied by a constant, every element in the matrix is enlarged by the constant.

  Thus, $c\begin{pmatrix}p & q\\r & s\end{pmatrix} = \begin{pmatrix}cp & cq\\cr & cs\end{pmatrix}$

- If A and B are any two matrices, AB = 0 does no necessarily imply that either A = 0 or B = 0.

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Example:

$$\text{If } A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

❒ Unlike ordinary algebra, the fact that AB = AC does not necessarily imply that B = C.

Example:

$$\text{If } A = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\text{then } AB = AC = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$$

But $B \neq C$ and $A \neq 0$

Example:

$$\text{Given } A = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 0 & -4 \\ 3 & -2 & 6 \end{pmatrix}$$

Solution:

Since A is 2 x 2 and B is 2 x 3, the product matrix AB is defined and is a 2 x 3 matrix.
To obtain the elements in the first row of the product matrix AB, multiply the first row (1 3) of A by the columns

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \end{pmatrix} \text{and} \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$ of B respectively.

$$\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 2 & 0 & -4 \\ 3 & -2 & 6 \end{pmatrix}$$

Step 1:

$$\Rightarrow \quad (1x2 + 3x3 \quad 1x0 - 3x(-2) \quad 1x(-4) + 3x6)$$

$$= \quad (11 \quad -6 \quad 14)$$

To obtain the elements in the second row of the product matrix AB, multiply the second row (2, -1) of A by the columns of B respectively.

$$\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 2 & 0 & -4 \\ 3 & -2 & 6 \end{pmatrix}$$

Step 2:

$$\Rightarrow \quad (\overset{11}{2x2 + (-1)x3} \quad \overset{-6}{2x0 + (-1)x(-2)} \quad \overset{14}{2x(-4) + (-1)x6})$$

$$= \quad \begin{pmatrix} 11 & -6 & 14 \\ 1 & 2 & -14 \end{pmatrix}$$

Matrix multiplication does not obey the commutative law, i.e. the products AB and BA of matrices need not be equal.

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9.4 Equivalent Matrices

Two matrices are equal if and only if their corresponding elements are equal. For instance, if

$A=\begin{pmatrix}2&3\\4&5\end{pmatrix}$, and $B=\begin{pmatrix}2&3\\4&5\end{pmatrix}$

then matrix A = matrix B.

Example:

Given $A = \begin{pmatrix}x&2\\1&y\end{pmatrix}$, $B=\begin{pmatrix}3&-5\\4&2\end{pmatrix}$ and $C=\begin{pmatrix}8&-3\\5&0\end{pmatrix}$

a. Find the values of x and y if A + B = C

b. Is BC = CB?

c. Evaluate 3B.

Solution:

a. $A+B = \begin{pmatrix}x&2\\1&y\end{pmatrix}+\begin{pmatrix}3&-5\\4&2\end{pmatrix}$

$= \begin{pmatrix}x+3&2-5\\1+4&y+2\end{pmatrix}$

$= \begin{pmatrix}x+3&-3\\5&y+2\end{pmatrix}$

Since A + B = C

$\Rightarrow \begin{pmatrix}x-3&-3\\5&y+2\end{pmatrix} = \begin{pmatrix}8&-3\\5&0\end{pmatrix}$

$\Rightarrow$ X-3 = 8 and y+2 =0

Therefore x=5, y=-2

b. $BC = \begin{pmatrix}3&-5\\4&2\end{pmatrix}\begin{pmatrix}8&-3\\5&0\end{pmatrix}$

$= \begin{pmatrix}24-25&-9+0\\32+10&-12+0\end{pmatrix} = \begin{pmatrix}-1&-9\\42&-12\end{pmatrix}$

$CB = \begin{pmatrix}8&-3\\5&0\end{pmatrix}\begin{pmatrix}3&-5\\4&2\end{pmatrix}$

$= \begin{pmatrix}24-12&-40-6\\15+0&-25+0\end{pmatrix}$

$= \begin{pmatrix}12&-46\\15&-25\end{pmatrix}$

Thus BC ≠ CB

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c. $3B = 3\begin{pmatrix} 3 & -5 \\ 4 & 2 \end{pmatrix}$

$= \begin{pmatrix} 3x3 & 3(-5) \\ 3x4 & 3x2 \end{pmatrix}$

$= \begin{pmatrix} 9 & -15 \\ 12 & 6 \end{pmatrix}$

Example: (A typical examination question)

A company makes two microcomputers, named SOLAR and GEM, using three main components X, Y and Z

SOLAR is made of 3 X components and 2Y components.
GEM is made of 2 X components and 3Z components,

i. Express this information as a 2 by 3 array Q.

The company receives an ORDER for 100 SOLAR and 50 GEM microcomputers.

ii. Express the ORDER as a row matrix (array) P.

iii. Find the product PQ and interpret its meaning.

A second order for 50 solar and 100 gem microcomputer is subsequently received but by then New Production Techniques have resulted in the time needed to produce X cut by 20%, and the time to produce Z cut by a third. The time taken to produce Y is unchanged.

vi. Use matrix (array) techniques to determine the COMPONENT REQUIREMENTS for the Second Order.

vii. Determine the HOURS needed to produce the SECOND ORDER.

For the FIRST ORDER labour costs were 3 units per hour, but had INCREASED by 10% for the SECOND ORDER.

viii. Determine whether these LABOUR COSTS are greater or smaller than those for the FIRST ORDER.

Solution:

i. Matrix Q:

ii. Matrix P:

	SOLAR	GEM
ORDER	(100	50)

iii. PQ is

	X	Y	Z
	(400	200	150)

This means that the ORDER requires 400 of X, 200 of Y and 150 of Z.

iv. R is a 3 x 1 matrix

$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

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v. PQR is a 1 x 1 matrix (1450)

This means that the *order* will require 1450 hours To produce (i.e. total time required to produce all X, Y and Z components to fulfill the order).

vi. A second order requires

$$(50\ 100)\begin{pmatrix}3 & 2 & 0\\ 2 & 0 & 3\end{pmatrix} = (350\ 100\ 300)$$

Hence it requires 350 of X, 100 of Y and 300 of Z.

vii. Hours to produce the second order is given by:

$$(350\ 100\ 300)\begin{pmatrix}1.6\\ 1\\ 2\end{pmatrix}$$

i.e. 560 + 100 + 600 = 1260 hours

viii. Labour costs for first order = 3 x 1450 = 4350
Labour costs fro second order = 3.3 x 126 = 4158

Hence the labour costs for the second order are smaller than those for the first order.

9.5 Transformations

A transformation is an operation which transform a point or a figure into another point or figure.

9.5.1 Translation

A translation is a transformation which moves all points in a plane through the same distance in the same direction.

E.g. The triangle ABC has been transformed onto the triangle A'B'C' by a translation $\begin{pmatrix}3\\ 2\end{pmatrix}$ i.e. 3 squares to the right and 2 squares up in the plane of the paper.

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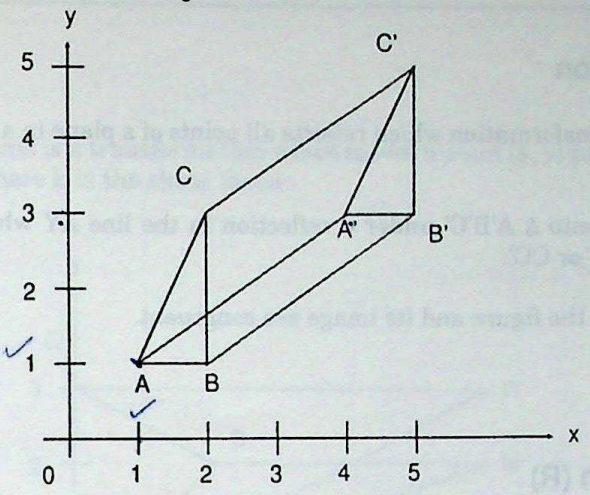


Point A is mapped onto A' by a translation $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$, denoted by T.

$$\begin{pmatrix} x \\ y \end{pmatrix} + T = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

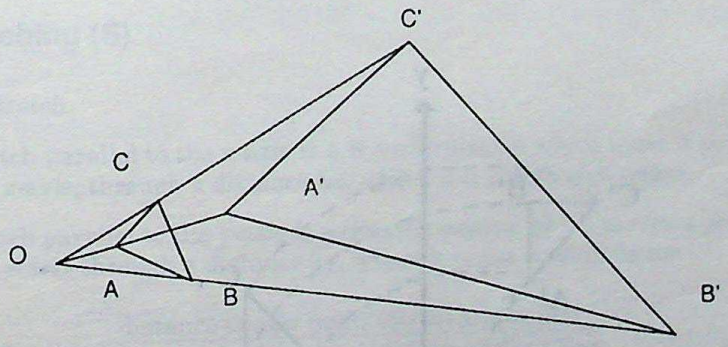
9.5.2 Enlargement (E)

An enlargement with centre O, scale factor k is a transformation which enlarges a given figure by k times the original size.

If $k > O$, the given figure and its image are on the same side of the centre of enlargement O.
If $k < O$, the given figure and its image are on opposite sides of O.

The figure and its image after an enlargement are similar. The scale factor

$$k = \frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC}$$



Under an enlargement, $\frac{\text{Area of Image}}{\text{Area of Figure}} = k^2$

If the image of a point (x, y) under a transformation is the point itself i.e. (x, y), the point (x, y) is called an invariant point of the transformation.

If a line is mapped onto itself under a transformation, the line is said to be an invariant line under the transformation.

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9.5.3 Reflection

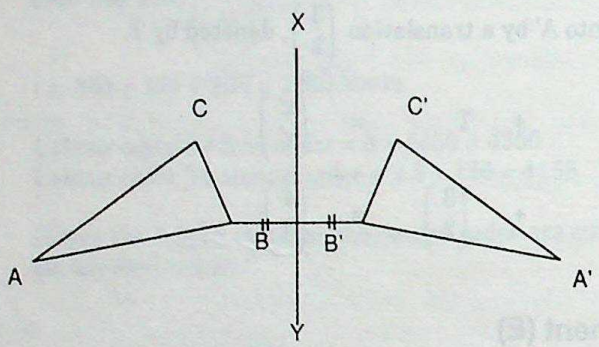
A reflection is a transformation which reflects all points of a plane in a line (on the plane) called the mirror line.

ΔABC is mapped onto Δ A'B'C' under a reflection in the line XY which is the perpendicular bisector of AA', BB' or CC'.

Under a reflection, the figure and its image are congruent.

Example:

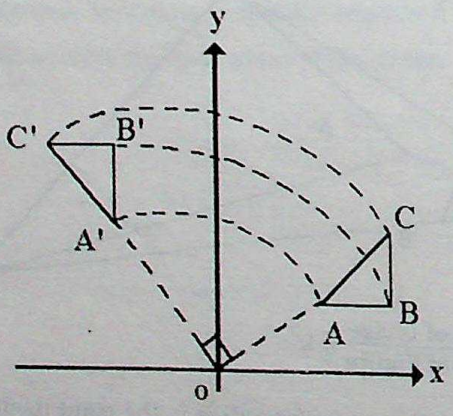
9.5.4 Rotation (R)



A rotation is a transformation which rotates all points on a plane about a fixed point known as the centre of rotation, through a given angle in anti-clockwise of clockwise direction.

The angle through which the points are rotated is called the angle of rotation.

The triangle ABC is rotated about the origin O through 90^{o} in the anti-clockwise direction, and mapped onto triangle A'B'C'.



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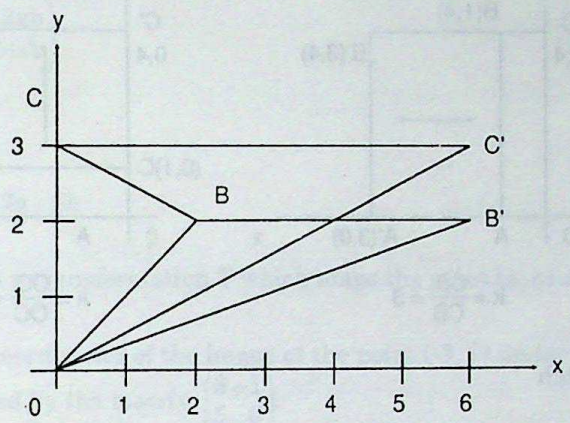
9.5.5 Shearing (H)

A shear parallel to the x-axis is a transformation which moves a point (x, y) parallel to the x-axis through a distance ky, where k is the shear factor.



$$k = \frac{\text{difference in x–coordinates of corresponding points}}{\text{y–coordinateof original point}}$$

ΔOBC is mapped onto $\Delta OB'C'$ under a shear along the x-axis with factor k.

$$k = \frac{CC'}{OC} = \frac{6}{3} = 2$$

A shear parallel to the y-axis is a transformation which moves a point (x, y) parallel to the y-axis through a distance kx where k is the shear factor.

$$k = \frac{\text{difference in y–coordinatesof corresponding points}}{\text{x–coordinateof original point}}$$

9.5.6 Stretching (S)

❒ One-way stretch.

- A stretch parallel to the x-axis is a transformation which move a point (x, y) parallel to the x-axis, through a distance kx, where k is the stretch factor.
- A stretch parallel to the y-axis is a transformation which moves a point (x, y) parallel to the y-axis through a distance ky, where k is the stretch factor.

$$k = \frac{\text{distance of new point the invariant line}}{\text{distance of original point the invariant line}}$$

In the case of stretching parallel to the x-axis, the invariant line is the x-axis.

In the case of stretching parallel to the y-axis, the invariant line is the y-axis.

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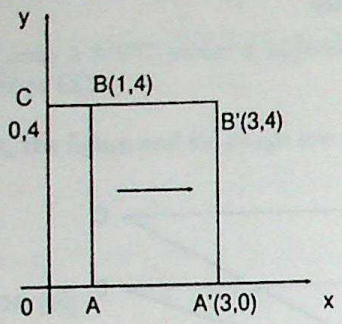
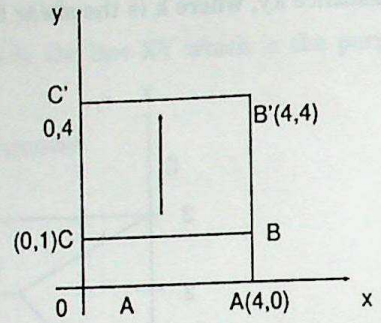
i.



$$k = \frac{CB'}{CB} = 3$$

ii.

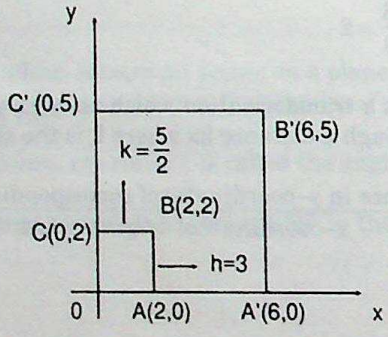


$$k = \frac{OC'}{OC} = 4$$

❒ Two Way Stretch

If a figure is stretched parallel to the x-axis as well as parallel to the y-axis, then the stretch is called a two-way stretch.

Under a two-way stretch with h and k as constants of stretch parallel to the x-axis and y-axis respectively a point (x, y) is mapped onto (hx, ky).



OABC → OA'B'C'
(x,y) → (hx,ky)

Example: Matrix $\begin{pmatrix} 1 & 3 \\ 2 & -5 \end{pmatrix}$ represents a transformation T.

Given (x, y) is the image of the point (a, b) under the transformation T, find x and y in terms of a and b.

Solution: Write the ordered pairs, (a, b) and (x, y) as column vectors:

$\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix}$

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Premultiply $\begin{pmatrix} a \\ b \end{pmatrix}$ by the matrix $\begin{pmatrix} 1 & 3 \\ 2 & -5 \end{pmatrix}$ we get

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$$

$$= \begin{pmatrix} 1xa + 3xb \\ 2xa + (-5)xb \end{pmatrix}$$

$$= \begin{pmatrix} a + 3b \\ 2a - 5b \end{pmatrix}$$

Therefore, $x = a + 3b$, $y = 2a - 5b$

The matrix $\begin{pmatrix} 1 & 3 \\ 2 & -5 \end{pmatrix}$ defines a transformation T which maps the point (a, b) onto $(a + 3b, 2a - 5b)$.

Example: Find the coordinates of the image of the point (-3, 2) under the transformation represented by the matrix $\begin{pmatrix} 3 & -1 \\ 5 & 0 \end{pmatrix}$.

Solution: Let the image of the point = (x, y)

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 5 & 0 \end{pmatrix}\begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3x(-3) + (-1)x2 \\ 5x(-3) + 0x2 \end{pmatrix}$$

$$= \begin{pmatrix} -9-2 \\ -15+0 \end{pmatrix}$$

$$= \begin{pmatrix} -11 \\ -15 \end{pmatrix}$$

Therefore the image of the point = (-11, -15)

Example: Find the matrix of the transformation which maps (1, 0) onto (4, 1) and (0, 1) onto (3, 2).

Solution: Let the matrix of transformation = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$(1, 0) \rightarrow (4, 1)$

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$ because (4, 1) is the image of (1, 0)

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} a+0 \\ c+0 \end{pmatrix}$$

$$= \begin{pmatrix} a \\ c \end{pmatrix}$$

Therefore $a = 4$, $c = 1$

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Points to Remember

- Definition of a matrix.
  - A matrix is a rectangle array of m rows and n columns.
- Order of matrix
  - A matrix is said to be of order m*n if it has m rows and n columns.
- Algebra of matrix
  - Addition / subtraction
    - This can be carried out provided the matrices are of the same order. The resultant matrix will have the same order as well.
  - Scalar product
    - A matrix can be multiplied by a constant (scalar). Each element in the matrix will be multiplied by that constant.
  - Product of matrices
    - This can be done provided the number of columns of the first matrix corresponds to to number of rows of the second matrix.

If $C = A * B$

Let $a_{i,j}$ denote the element in matrix A,
$b_{m,n}$ denote the element in matrix B,
and $c_{p,q}$ denote the element in matrix C
where (i, j), (m, n) and (p, q) denote the row and column number respectively.

Then

$$C_{p,q} = \sum_{r=1}^{N} a_{p,r} * b_{r,q}$$

where N = number of columns in matrix A
= number of rows in matrix B

For example,

$C_{3,2} = a_{3,1}\, b_{1,2} + a_{3,2}\, b_{2,2} + a_{3,3}\, b_{3,2} + \ldots + a_{3,N}\, b_{N,2}$

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❒ Transformations:

Table giving matrices corresponding to transformations:

Transformations	Matrices	Matrix Equation
Translation	$\begin{pmatrix} h \\ k \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} h \\ k \end{pmatrix}$
Enlargement with centre at origin and scale factor k.	$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
Reflection in x-axis	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
Reflection in y-axis	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
Reflection in line y=x	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
Reflection in line y=-x	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
Rotation of 90° anti-clockwise about origin	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
Rotation of 90° clockwise about origin	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

Transformations	Matrices	Matrix Equation
Rotation of 180 about origin	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
Shear parallel to x-axis, shear factor k	$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
Shear parallel to y-axis, shear factor h	$\begin{pmatrix} 1 & 0 \\ h & 1 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ h & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
Stretch parallel to x-axis, stretch factor k	$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
Stretch parallel to y-axis, stretch factor h	$\begin{pmatrix} 1 & 0 \\ 0 & h \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & h \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
Stretch parallel to x-axis and y-axis with factors k and h respectively	$\begin{pmatrix} h & 0 \\ 0 & k \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} h & 0 \\ 0 & k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

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Past Years Questions

1. Given $A = \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}$, $B = [-4 \;\; 1]$ and $C = \begin{bmatrix} 7 \\ 6 \\ -6 \end{bmatrix}$ evaluate

   a. $A * B$ [3]

   b. $A - C$ [2]

2. Given:

   $D = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $Z = [2 \; 1 \; 3]$ $C = \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}$

   Calculate, showing all working

   a. $D * Z$ [2]

   b. $Z * D$ [1]

   c. $D - C$ [1]

3. Let $A = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

   a. Evaluate $A * B$ [2]

   b. Evaluate $B * A$ [2]

   c. Comment on the results from a. and b. [2]

4. Given $A = \begin{bmatrix} -1 & 5 \\ 0 & 3 \\ 6 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 2 & 0 \\ 0 & 3 & 4 \end{bmatrix}$

   a. Calculate $A * B$ [3]

   b. Given $C = \begin{bmatrix} -5 & 12 & -6 \\ 1 & 3 & -1 \end{bmatrix}$, evaluate $2B - C$ [3]

5. Calculate $X = 2AB$,

   given $A = \begin{bmatrix} 2 & -2 \\ 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 2 \\ 1 & 3 \end{bmatrix}$ [3]

6. Given $A = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 5 & 4 \\ 1 & 8 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 & -1 \\ 2 & 1 & 3 \\ 1 & -2 & 0 \end{bmatrix}$

   Prove that $AB \neq BA$. [6]

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7. Indicate whether each of the following statements concerning matrices are True or False:

a. If A + B = C, then B + A = C [1]

b. If A * B = B * A, then A = B [1]

c. It is always possible to add together matrices which can be multiplied together [1]

d. For square matrices, it is often possible to show that $A * B \neq B * A$ [1]

e. Which of the following is not a valid matrix equation? [1]

i. A / B = C

ii. A - B = 2(B - A)

iii. A * B = B * A + C [1]

8. a. i. Use matrices $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ to calculate 2A * B [4]

ii. Calculate A - B [2]

iii. Given matrix C = [4 8], calculate C * 2A [4]

b. i. Three types of personal computer A, B and C sell at $3,000, $4,000 and $5,000 respectively. Express these information as 3x1 matrix and label it X. [2]

ii. Customer 1 wishes to buy 3 type A, 4 type B and 3 type C. Customer 2 wishes to buy 1 type A, 2 type B and 5 type C. Express this information as a 2x3 matrix and label it Y. [3]

iii. Calculate YX and interpret the answer [5]

9. a. A student scores 75%, 85%, 60% and 43% for the subjects Maths, English, French and Science respectively. The Full mark for each subject is as follow :

Maths	-	100
English	-	200
French	-	150
Science	-	150

i. Arrange the full mark in a 1 * 4 array and label it as Z. [2]

ii. Arrange the scores in a column array and label it as W. [2]

iii. Find the total mark obtained by the student using W and Z. [4]

iv. From the matrix determine the student's English and Science marks. [2]

b. The following reductions were made, after moderation, from the existing scores 75%, 85%, 60% and 43%:

Maths	-	5%
English	-	10%
French	-	8%
Science	-	3%

i. Arrange the new scores in a column array and label it as Y. [2]

ii. Find the new total obtained by the student. [4]

iii. What is the difference in percentage between the new and old total marks. [4]

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10. a. Using matrices $A = \begin{bmatrix} -1 & 5 & 0 \\ 0 & -2 & 3 \end{bmatrix}$, and $B = \begin{bmatrix} -8 & 1 \\ 2 & 4 \\ -4 & 1 \end{bmatrix}$

Calculate AB [4]

b. There are three types of printer : S for serial printers, L for line printers, and P for page printers. They cost \$250, \$500 and \$1500 respectively.

i. Express the information above as a row matrix and label it as X. [2]

ii. Customer A wishes to buy 10 of S, 5 of L and 2 of P while customer B buys 20 of S, 3 of L and 4 of P. Express the following information as a matrix of dimension 3x2. Label it as Y. [2]

iii. Determine an appropriate matrix R derived from the calculation of X and Y to represent the total cost to each customer. [5]

iv. Special discounts are given, 5% to customer A and 10% to customer B. Express this information as a column matrix. [2]

v. Using matrix calculation only, determine the total of the actual costs paid by both customers. [5]

11. A company receives bills for rental, heating and lighting twice a year. If i = 1,2 represents the first and the second half of the year and j = 1, 2, 3 represents the Rental, Heating and Lighting, then the matrix P whose entries are identified as $P_{i,j}$ is used to represent the half-yearly bills.

a. Explain in words, the meaning of $P_{2,1} = \$1050$ [2]

b. If $P = \begin{bmatrix} 3000 & 1550 & 1600 \\ 1550 & 2050 & 1800 \end{bmatrix}$, using matrix

calculation determine a matrix R which represents the total expense for Rental, Heating and Lighting, respectively for each half of the year.Q [4]

c. If 5% discount is given for Rental, 10% for Heating and 5% for Lighting, represent this information as a 3x1 matrix Q. [2]

d. Calculate S = PQ [4]

e. What does this matrix S represent? [2]

f. Calculate [1 1]*(R - S). Interpret your answer. [4]

g. What is the expense for heating in the second half of the year, after discount? [2]

12. The Computa Co. buys three types of computer. the SL, the SLX and the SLXi.

a. The purchase price is \$750, \$850 & \$900 respectively. Express this as a 1 x 3 matrix, P. [1]

b. The Computa Co. sells them at \$815, \$935 & \$1,000. Express this as a 1 x 3 matrix, S. [1]

c. In a certain month, The Computa Co. makes the following sales of each type:

First week, 10, 5 and 2. Second week 8, 3 and 1.
Third week, 8, 6 and 0. Fourth week 6, 2 and 2.

Express the sales figures as a 3 x 4 matrix, F. [3]

d. If $Z = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ Calculate, [(S * F) - (P * F)] * Z [4]

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e. Calculate the percentage profit, correct to 1D. [2]

13. For $-3 \le x \le 5$ and $-5 \le y \le 5$, and a scale of 2cm : 1 unit on both axes.

a. i. Plot the points P(5, -3), Q(4, -2), R(3, -2) and S(2, -3) ON GRAPH PAPER. [3]

ii. Join P to Q, Q to R, R to S and S to P. Label this as shape A. [1]

b. i. Write down the matrix representation of shape A. [1]

ii. If $T1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $T2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $B = T_1 * A$, $C = T_2 * A$.

Calculate matrix B and matrix C. [4]

c. Plot, draw and LABEL shapes B and C on the same sheet of graph paper as shape A.

d. i. Calculate the matrix D, where $D = T_1 * C$. [4]

ii. Plot, draw and LABEL shape D on the same sheet of graph paper as shapes A, B and C. [2]

e. Describe fully the SINGLE transformation that move shape A to: [6]

i. Shape B

ii. Shape C

iii. Shape D

14. Using a scale of 2 cm = 1 unit on both axes for $-5 \le x \le 4$ and $-5 \le y \le 5$

a. i. Plot the points P(-5, -1), Q(-2, 0) and R(-3, 2) on GRAPH PAPER. [3]

ii. Using straight lines, join P to Q, Q to R and R to P. LABEL this as shape A. [1]

b. i. Write down the matrix representation of shape A. [1]

ii. If $T1 = \begin{bmatrix} 8 & 8 & 8 \\ 1 & 1 & 1 \end{bmatrix}$ and $T_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $B = T_1 + 2 * A$ and $C = T_2 * A$.

Calculate matrix B and matrix C. [4]

c. Plot, draw and LABEL shape B and C on the same sheet of graph paper as shape A. [2]

d. i. Calculate the matrix D, when $D = T_2 * B$. [2]

ii. Plot, draw and LABEL shape D on the same sheet of graph paper. [2]

e. If $D = 2 * C + E$, calculate the matrix E. [3]

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15. a. ON GRAPH PAPER, using a scale of 1 cm : 1 unit on both axes

i. Plot the point P(2, 1) [1]

ii. Draw the three vectors, $PQ = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$QR = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ $RS = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$ [4]

iii. Draw and write the resultant of PQ + QR + RS as a column vector. [2]

b. Label the quadrilateral P Q R S as shape A and write down its representation as a 2 by 4 matrix. [2]

c. If $T_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $T_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $B = T_1 * A - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 \end{bmatrix}$ and $C = T_2 * B$.

Calculate matrix B and matrix C [5]

d. Plot, draw and LABEL shapes B and C on the SAME GRAPH PAPER as shape A. [2]

e. Describe fully the single transformation that moves shape A to:

i. shape B

ii. shape C [4]

16. On GRAPH PAPER, using a scale 1 cm = 1 unit on both axes for $-8 < x < 10$ and $-10 < y < 12$,

a. i. plot the points P(-4, 3), Q(0, 6), R(-5, 9), S(-3, 5) and T(-6, 5) [3]

ii. using STRAIGHT lines, join P to Q, Q to R, R to S, S to T and T to P. Label this shape A. [1]

iii. write down a 2 by 5 matrix representation of shape A. [1]

b. If $T_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $T_2 = \begin{bmatrix} 3 & 3 & 3 & 3 \\ 2 & 2 & 2 & 2 \end{bmatrix}$ and $B = T_1 * A$ and $C = 2 * B - 4 * T_2$;

calculate matrix B and matrix C. [5]

c. Plot, draw and LABEL shapes B and C on the same sheet of graph paper as shape A. [4]

d. If $D = T1 * C + 2 * A$; calculate matrix D. [4]

e. What would be the effect on any of the shapes if D was added? [2]

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17. a. A company sells three products, X, Y and Z at \$72, \$56 and \$67 respectively and sells 109 of X, 147 of Y and 100 of Z in one week and 120 of X, 185 of Y and 94 of Z in the next week.

i. Put the two-weeks quantities in a 3 x 2 matrix, N and the prices in a 1 x 3 matrix, S [2]

ii. If R = SN find the matrix R. [2]

Products Y and Z cost \$n each to produce X costs twice as much as either of the others;

iii. Express this information as 1 x 3 matrix, C [1]

iv. If E = CN find the matrix E [2]

If P = R - E find the matrix P and state what is represents [2]

vi. If the total profit in week 2 is 10% more than in week 1 find \$n. [1]

18. a. A company receives its bills for heating, lighting and rent four times a year. If i = 1, 2, 3, 4 represents the FOUR QUARTERS in a year and j = 1, 2 or 3 represents HEATING, LIGHTING or RENT, then the matrix P whose entries are identified by $P_{i,j}$ is used to represent the quarterly bills.

i. Explain in words, the meaning of $P_{3,2} = \$1800$ [2]

ii. If for one full year, $P = \begin{bmatrix} 2500 & 1650 & 2000 \\ 2100 & 1400 & 2200 \\ 2150 & 1100 & 2200 \\ 2260 & 1350 & 2400 \end{bmatrix}$

What is the total of $P_{i,1}$ for i = 1, 2, 3 and 4? [1]

iii. Explain in words what the total represents. [2]

b. The 3 x 1 matrix Q, represents in DECIMAL FRACTION form, discounts received by the company of 10% on heating, 5% on lighting and 0% on rent

i. Write down the matrix Q [1]

ii. Calculate the matrix R if $R = P \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ [4]

iii. Calculate the matrix S, if S = P x Q [4]

iv. What does the matrix S represent? [2]

v. Calculate [1 1 1 1] x (R - S). What does this represent? [3]

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19. a. Shape A is represented by the matrix

$$\begin{bmatrix} 0 & 2 & 5 & 4 & 3 & 2 \\ -4 & -2 & -4 & -3 & 2 & 4 \end{bmatrix}$$

i. Calculate

$$B = \begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 \\ -3 & -3 & -3 & -3 & -3 & -3 \end{bmatrix} + 2 * A$$ [2]

ii. Calculate $C = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} * \left(\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} * A \right)$ [4]

b. By using a SCALE of 1 cm = 1 unit on both axes, PLOT and DRAW the shapes A, B and C on one sheet of GRAPH PAPER. [10]

c. i. How are the shapes B and A related? [1]

ii. What TRANSFORMS have been made to A to obtain C? [2]

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Chapter 10

Statistics

Chapter Objectives

At the completion of this chapter, you would have learnt:

- to understand why statistic are used to solve real life problem;
- to use general guidelines to organize data into Frequency Table;
- to use various charts and graph to display data, e.g. Histogram Cumulative Frequency Diagram;
- to calculate Standard Deviation, Variance as measures of central tendency;
- to calculate Standard Deviation, Variance as measures of spread or dispersion.

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10.1 Introduction

An addition to our 'ordinary; Boolean algebra, De Morgan's Laws is one tool used for simplifying complex expressions in Boolean algebra or complex switching circuits.

- Process that entails destruction of products if physical testing is carried.
  - E.g. To substantiate the life span of light bulbs from a factory.
- Situations are too large and too complicated if physically counting were carried out.
  - E.g. The number of people suffering from AIDS in Asia.
- Situations where forecast or predictions are to be made based on past information.
  - E.g. The weather forecasting.

Statistics involve the process of collecting data from a sample, make appropriate deductions from the sample. In this chapter, we will look at now to organize data into frequency table, display data in proper charts and calculate relevant quantities.

10.2 Raw Data

Raw data, once collected has to be organized numerically. An example is the set of names of male students obtained from an alphabetical listing of a public school records.

10.2.1 Arrays

An array is an arrangement of raw numerical data in ascending or descending order of magnitude. The difference between the largest and smallest numbers is called the range of the data. For example, if the heaviest weight of 100 male students is 74 kg and the lightest weight is 60 kg, then the range is 74-60 which gives 14kg.

10.3 Grouped Data

10.3.1 Frequency Distributions

When summarizing large masses of raw data it is often useful to distribute the data into classes or categories and to determine the number of individuals belonging to each class, called the class frequency. A tabular arrangement of these data by classes together with the corresponding class frequencies is called a frequency distribution or frequency table. The table below shows a frequency distribution of weights (recorded to the nearest kg) of 100 male students at Informatics Computer School.

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Mass (Kilogram)	Number of Students
60 - 62	5
63 - 65	18
66 - 68	42
69 - 71	27
72 - 74	8
	Total = 100

The first class, for example, consists of weights from 60 to 62kg. Since 5 students have weights falling between this class, the corresponding class frequency is 5.

Data organized and summarized as in the above frequency distribution are often called grouped data.

10.3.2 Class Intervals and Class Limits

A range of values defining a class such as 60-62 in the above table is called a class interval. The end numbers, 60 and 62, are called class limits. The smallest number 60 is the lower limit and the larger number 62 is the upper class limit.

10.3.3 Class Boundaries

If weights are recorded to the nearest kg, the class interval 60-62 theoretically includes all measurements from 59.5 kg to 62.5kg. These numbers, indicated briefly by the exact numbers 59.5 and 62.5, are called class boundaries or true class limits. The smaller number 59.5 is the lower class boundary and the larger number 62.5 is the upper class boundary.

Sometimes, class boundaries are used to symbolize classes. For example, the various classes in the first column of the previous table could be indicated by 59.5-62.5, 62.5-65.5, etc.

10.3.4 The Size of a Class Interval

The size of a class interval is the difference between the lower and the upper class boundaries and is also referred to as the class width, class size or class strength. For instance, the class interval for the above example is 62.5 - 59.5 = 3.

10.3.5 The Class Mark

The class mark is the midpoint of the class interval and is obtained by adding the lower and upper class limits and dividing by two. Thus the class mark of the interval 60 -62 is $\frac{(60+62)}{2} = 61$. The class mark is also known as the class midpoint.

❒ General rules for forming frequency distributions

- Determine the largest and smallest numbers in the raw data and thus find the range.
- Divide the range into a convenient number of class intervals having the same size.
- Determine the number of observations falling into each class interval.

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10.4 Presentation of Statistical Data

A graph is a pictorial representation of the relationship between variables. Many types of graphs are employed in statistics, depending on the nature of the data involved and the purpose for which the graphs are intended. Among these are bar graphs, pie graphs, pictographs, etc. These graphs are sometimes referred to as charts or diagrams.

10.4.1 Histogram and Frequency Polygons

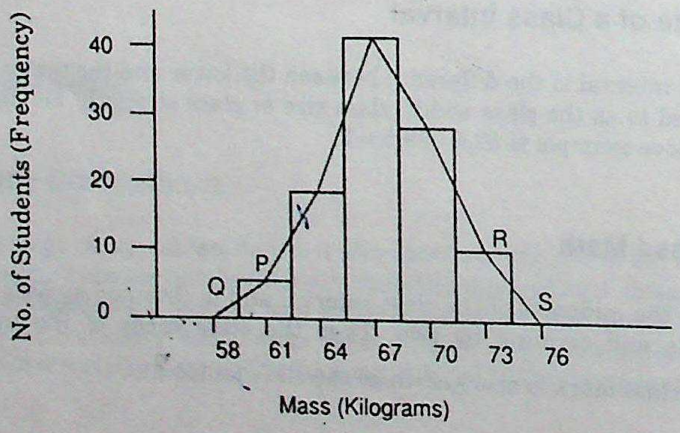
There are two graphical representations of frequency distributions.

- A histogram of a frequency distribution consists of a set of rectangles having
  - Bases on a horizontal axis with centres at the class marks and lengths equal to the class interval sizes; and
  - Areas proportional to class frequencies.
- A frequency polygon corresponding to the above frequencies plotted against class marks. It can be obtained by connecting midpoints of the tops of the rectangles in the histogram.

The histogram and frequency polygon corresponding to the above frequency distribution of weights are shown on the same set of axes in the graph below. It is necessary to add the extensions PQ and RS to the next lower and higher class marks which have corresponding class frequency of zero.

10.4.2 Cumulative Frequency Distributions

The total frequency of all values less than the upper class boundary of a given class interval is called the cumulative frequency up to and including the class interval. For example, the cumulative frequency up to and including the class interval 66-68 is 5 + 18 + 42 = 65, signifying that altogether 65 students have weights less than 68.5kg.



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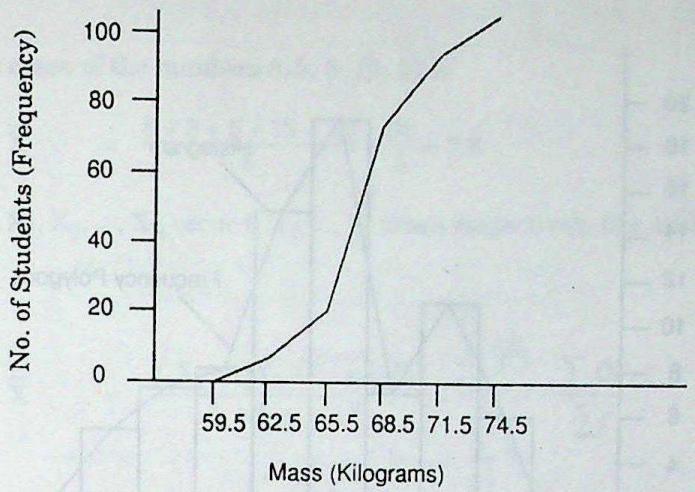

A graph showing the cumulative frequency less than any upper class boundary plotted against the upper class boundary is called a cumulative frequency polygon. (Ogive)

Example:

The final marks for Computer Science of 80 students at ABC University are recorded in the following table.

68 84 75 82 68 90 62 88 76 93
73 79 88 73 60 93 71 59 85 75
61 65 75 87 74 62 95 78 63 72
66 78 82 75 94 77 69 74 68 60
96 78 89 61 75 95 60 79 83 71
79 62 67 97 78 85 76 65 71 75
65 80 73 57 88 78 62 76 53 74
86 67 73 81 72 63 76 75 85 77

Using the above data, draw:

a. A histogram,

b. A frequency polygon, and

c. A cumulative frequency curve.

Solution:

Class	Class Mark	Frequency	Cumulative Frequency
56 - 60	58	6	6
61 - 65	63	11	17
66 - 70	68	7	24
71 - 75	73	19	43
76 - 80	78	15	58
81 - 85	83	8	66
86 - 90	88	7	73
91 - 95	93	5	78
96 - 100	98	2	80

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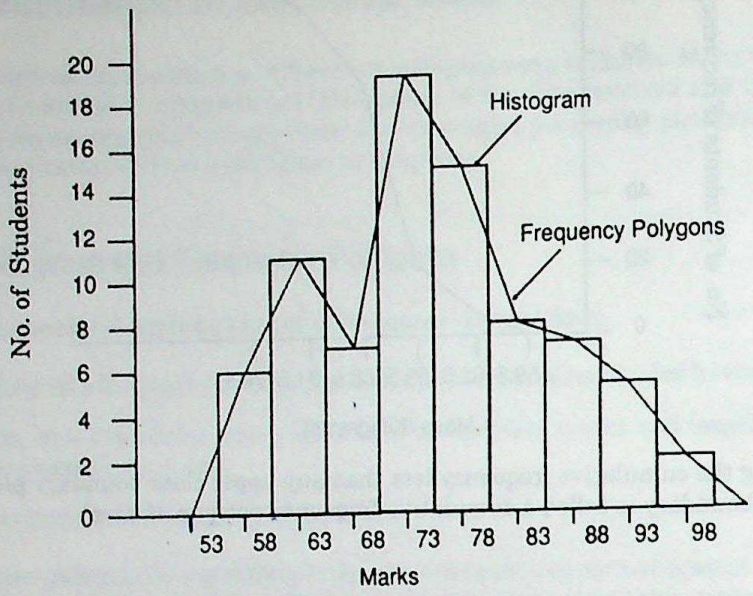

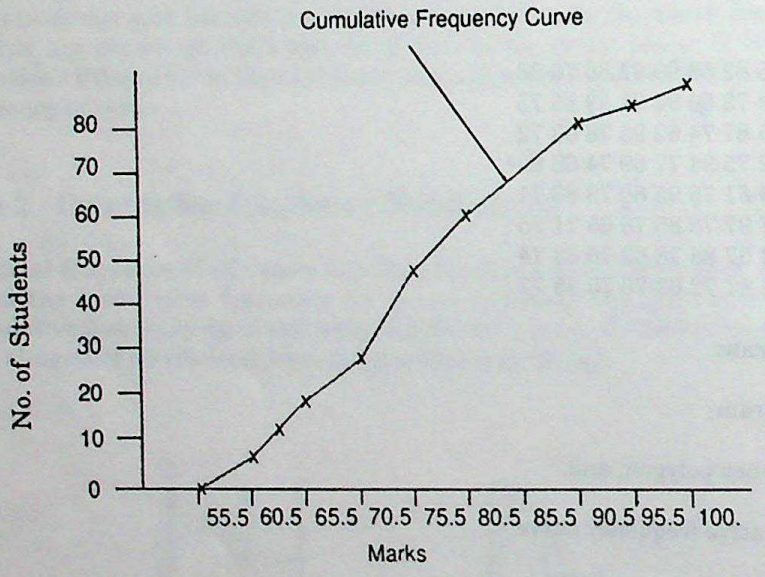

10.5 Three Statistical Quantities of Central Tendency

10.5.1 The Arithmetic Mean

The arithmetic mean or the mean of a set of n numbers $X_1, X_2, X_3, ..., X_N$ is denoted by $\overline{X}$ and is defined as

$$\overline{X} = \frac{X_1 + X_2 + X_3 + \ldots + X_N}{N} = \frac{\sum_{j=1}^{N} X_j}{N} = \frac{\sum X}{N}$$

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Example:

The arithmetic mean of the numbers 8, 3, 5, 12, 10 is

$$\overline{X} = \frac{8 + 3 + 5 + 12 + 10}{5} = \frac{38}{5} = 7.6$$

If the numbers $X_1, X_2, ..., X_N$ occur $f_1, f_2, ..., f_N$ times respectively the arithmetic mean is given by :

$$\overline{X} = \frac{f_1X_1 + f_2X_2 + \ldots + f_NX_N}{f_1 + f_2 + \ldots + f_N} = \frac{\sum_{j=1}^{N} f_jX_j}{\sum_{j=1}^{N} f_j} = \frac{\sum fX}{\sum f}$$

Example:

The arithmetic mean of the numbers 5, 8, 6 and 2 which occurs 3, 2, 4 and 1 time respectively is :

$$\overline{X} = \frac{(3)(5) + (2)(8) + (4)(6) + (1)(2)}{3 + 2 + 4 + 1} = \frac{15 + 16 + 24 + 2}{10} = 5.7$$

10.5.2 The Median

The median of a set of numbers arranged in order of magnitude (i.e. in an array) is the middle value or the arithmetic mean of the two middle values.

Example:

The set of numbers 3, 4, 4, 6, 6, 8, 8, 8, 10 has a median of 6.

Example:

The set of numbers 5, 5, 7, 9, 11, 12, 15, 18 has a median = $\frac{1}{2}(9 + 11) = 10$.

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For a grouped data, the median, obtained by interpolation is given by

$$\text{Median} = L_1 + \left(\frac{\frac{N}{2} - (\sum f)_1}{f_{median}} \right) \times c$$

where

L1	=	Lower class boundary of the median class (i.e. the class containing the median).
N	=	Number of items in the data (i.e. total frequency).
$(\sum f)_1$	=	Sum of frequencies of all classes lower than the median class.
f_{median}	=	Frequency of median class.
c	=	Size of median class interval.

Median of grouped data may also be obtained graphically using cumulative Frequency Diagram.

Example:

By first creating a cumulative frequency table and then a cumulative frequency diagram, estimate the median of the following survey of the examination marks of 80 students on a particular computer course.

Marks (%)	No. of students
0 - 20	3
21 - 40	19
41 - 60	35
61 - 80	22
81 - 100	1

Median is the mark below which 50% of the students score, and therefore above which another 50% of the student score.

Marks (less than or =)	Cumulative Frequency
20	3
40	22
60	57
80	79
100	80

Cumulative Frequency Table

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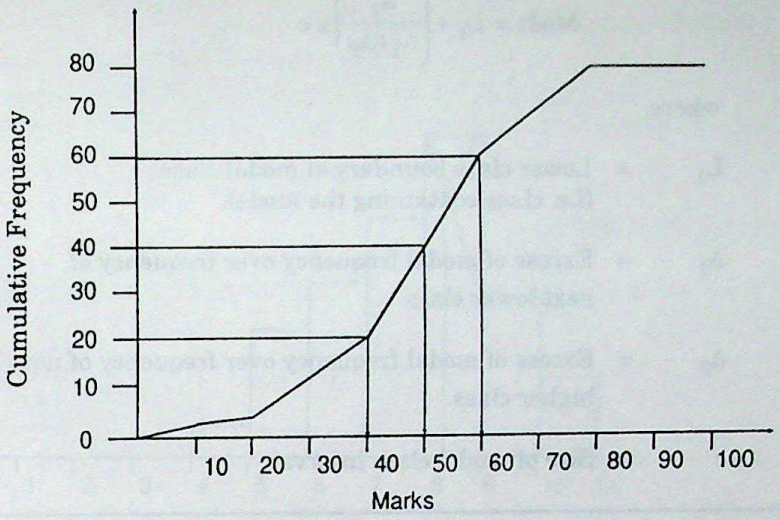

Figure 10-1

From the above Figure 8-1, there are 40 students who score 52 marks or less and the other 40 score more than 52 marks. The median is 52 marks.

In Fig 8, 39 mark is the *lower quartile*, which is the mark below which 25% of the population of students score (or 20 out of 80). 61 marks is the *upper quartile* below which 75% of the student score, (or 60 out of 80). The range of (61 - 39) = 22 is the Inter-quartile range.

10.5.3 The Mode

The mode of a set of number is that value which occurs with the greatest frequency, i.e. it is the most common value. The mode may not exist, and even if it does exist, it may not be unique.

Example:

The set 2, 2, 5, 7, 9, 9, 9, 10, 10, 11, 12, 18 has a mode = 9.

Example:

The set 3, 5, 8, 10, 12, 15, 16 has no mode.

A distribution having only one mode is called uni-modal.

In the case of a grouped data where a frequency curve has been constructed to fit the data, the mode will be the value (values) of X with the highest frequency.

From a frequency distribution or histogram the mode can be obtained using the following formula.

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$$\text{Mode} = L_1 + \left(\frac{\Delta_1}{\Delta_1+\Delta_2}\right) \times c$$

where

L_1 = Lower class boundary of modal class (i.e. class containing the mode).

Δ_1 = Excess of modal frequency over frequency of next lower class.

Δ_2 = Excess of modal frequency over frequency of next higher class.

c = Size of modal class interval.

Mode of grouped data may be estimated from Histogram as shown in Figure 10-1.

Example:

Estimate the mode of the following distribution of salaries of employees in a computer company:

Salary ($)	No. of people
4000 -	2
5000 -	12
6000 -	19
7000 -	25
8000 -	36
9000 -	17
10 000 -	9

a. By graphical means; and

b. By calculation means.

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Solution:

a.

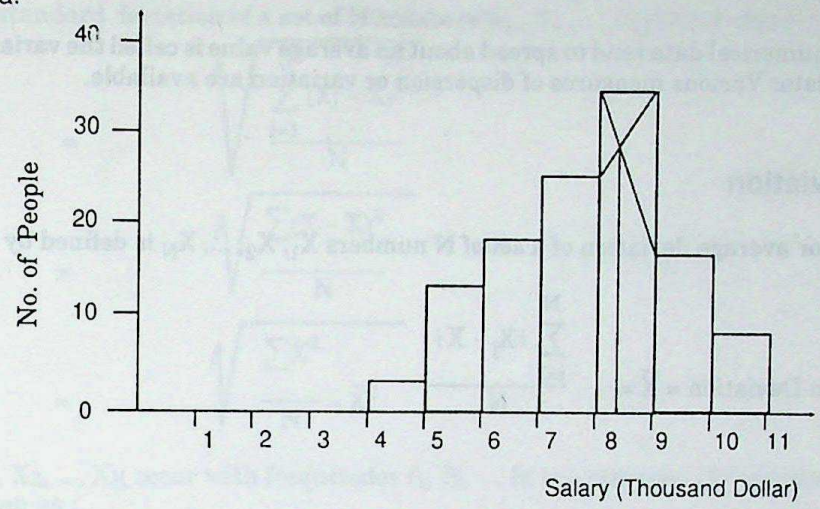

Figure 10-2:
Histogram

The category 8000 - 9000 has the highest frequency, it is called the *modal class*. The estimated *mode* is in this category (or class). It can be estimated as shown in Figure 8-2.

b. Mode can be estimated using the formula

$$\text{Mode} = L_1 + \left(\frac{\Delta_1}{\Delta_1+\Delta_2}\right) \text{x } c$$

$L_1 = 8000$

$\Delta_1 = 11$

$\Delta_2 = 19$

$c = 1000$

$$\text{Mode} = 8000 + \frac{11}{11 + 19} \text{ x } 1000 = 8366.67$$

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10.6 Dispersion and Variation

The degree to which numerical data tend to spread about an average value is called the variation or dispersion of the data. Various measures of dispersion or variation are available.

10.6.1 Mean Deviation

The mean deviation or average deviation of a set of N numbers $X_1, X_2, ..., X_N$ is defined by :

$$\text{Mean Deviation} = \bar{X} = \frac{\sum_{j=1}^{N} |X_j - X|}{N}$$

Example:

Find the mean deviation of the set of numbers 2, 3, 6, 8, 11.

Solution:

$$\text{Arithmetic Mean} = X = \frac{2+3+6+8+11}{5} = 6$$

$$\text{Mean Deviation (MD)} = \frac{|2-6| + |3-6| + |6-6| + |8-6| + |11-6|}{5}$$

$$= \frac{|-4| + |-3| + |0| + |2| + |5|}{5}$$

$$= \frac{4+3+0+2+5}{5}$$

$$= 2.8$$

If $X_1, X_2, ..., X_K$ occur with frequencies $f_1, f_2, f_3, ..., f_k$ respectively, the mean deviation can be written as :

$$\text{Mean Deviation} = \frac{\sum_{j=1}^{K} f_j |X_j - X|}{N}$$

where $N = \sum_{j=1}^{K} f_j = \sum f$

This form is useful for grouped data where the $X_{j's}$ represent class marks and the $f_{j's}$ are the corresponding frequencies.

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10.6.2 The Standard Deviation

The standard deviation of a set of N numbers $X_1, X_2, ..., X_K$ is denoted by SD and is defined by

$$SD = \sqrt{\frac{\sum_{i=1}^{N}(X_j - \bar{X})^2}{N}}$$

$$= \sqrt{\frac{\sum(X - \bar{X})^2}{N}}$$

$$= \sqrt{\frac{\sum X^2}{N} - \bar{X}^2}$$

If $X_1, X_2, ..., X_K$ occur with frequencies $f_1, f_2, ..., f_K$ respectively, the standard deviation can be written as :

$$SD = \sqrt{\frac{\sum_{i=1}^{K} f_i(X_j - \bar{X})^2}{N}} = \sqrt{\frac{\sum f(X - \bar{X})^2}{N}}$$

$$= \sqrt{\frac{\sum f X^2}{N} - \bar{X}^2}$$

where $N = \sum_{i=1}^{K} f_i = \sum f$

This form is useful for grouped data.

10.6.3 The Variance

The variance of a set N numbers $X_1, X_2, ..., X_K$ is defined as the square of the standard deviation and is thus given by

$$Var = \frac{\sum_{i=1}^{N}(X_j - \bar{X})^2}{N} = \frac{\sum(X - \bar{X})^2}{N} = \frac{\sum X^2}{N} - \bar{X}^2$$

If $X_1, X_2, ..., X_K$ occurs with frequencies $f_1, f_2, ..., f_K$ respectively, the variance can be written as

$$Var = \frac{\sum_{i=1}^{K} f_i(X_j - \bar{X})^2}{N} = \frac{\sum f(X - \bar{X})^2}{N} = \frac{\sum f X^2}{N} - \bar{X}^2$$

where $N = \sum_{i=1}^{K} f_i = \sum f$

This form is useful for grouped data.

Example:

Find the standard deviation and variance of the following set of numbers:

12, 6, 7, 3, 15, 10, 18, 5

$$\text{Arithmetic mean} = \bar{X} = \frac{\sum X}{N}$$

$$= \frac{12 + 6 + 7 + 3 + 15 + 10 + 18 + 5}{8}$$

$$= \frac{76}{8}$$

$$= 9.5$$

$$\text{SD} = \sqrt{\frac{\sum (X - \bar{X})^2}{N}}$$

$$= \sqrt{\frac{(12-9.5)^2 + (6-9.5)^2 + (7-9.5)^2 + (3-9.5)^2 + (15-9.5)^2 + (10-9.5)^2 + (18-9.5)^2 + (5-9.5)^2}{8}}$$

$$= \sqrt{23.75}$$

$$= 4.87$$

$$\text{Var} = (4.872)^2$$

$$= 23.75$$

Example:

Find the standard deviation of the weights of the 100 male students at Informatics Computer School as shown in the table below.

Mass (Kilogram)	Number of Students
60 - 62	5
63 - 65	18
66 - 68	42
69 - 71	27
72 - 74	8
Total	100

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Solution:

Mass (Kg)	Class Mark	Frequency (f)	fX	fX^2
60 - 62	61	5	305	18605
63 - 65	64	18	1152	73728
66 - 68	67	42	2814	188538
69 - 71	70	27	1890	132300
72 - 74	73	8	584	42632
		$N = \sum f = 100$	$\sum fX = 6745$	$\sum fX^2 = 455803$

$$\overline{X} = \frac{\sum fX}{\sum f} = \frac{\sum fx}{N} = \frac{6745}{100} = 67.45\text{kg}$$

$$SD = \sqrt{\frac{\sum fX^2}{N} - \overline{X}^2}$$

$$= \sqrt{\frac{455803}{100} - (67.45)^2}$$

$$= 2.92\text{kg}$$

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Points to Remember

- To use some guidelines to organize data into *frequency distribution table* (grouped data).
- Use *line graph, bar chart, piotogram, pie chart, histogram and cumulative frequency diagram* to display data in graphical form.
- Calculate *mean, median and mode* as measures of *central tendency*.

	Raw Data	Grouped Data
Mean	$\frac{\sum x}{N}$	$\frac{\sum fx}{\sum f}$
Median	Take the middle data from a sorted list.	1. Use cumulative frequency Diagram. 2. Use formula.
Mode	Take the data that appear most frequency.	1. Use histogram. 2. Use formula.

- Calculate *standard deviation* and *variance* as measures of *spread* or *dispersion*.

	Raw Data	Grouped Data
Standard Deviation	$\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2$	$\frac{\sum f(x)^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2$
Variance	(Standard Deviation)^2	(Standard Deviation)^2

- Use time series graph to observe the trend of a variable over time.
- Use scatter diagram to observe the correlation between two variables.

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Past Years Questions

1. Given the following collection of numbers 2, 4, 5, 5, 6, 7
   a. Calculate the mean (round to 1 decimal place) [2]
   b. Calculate the mode [1]
   c. Calculate the median [1]

2. a. What is mean? [1]
   b. What is mode? [1]

3. A grade of 1 to 5 could be obtained in an examination and the actual scores were distribute as follows :

Grade	1	2	3	4	5
No. of Candidates	4	3	9	2	2

   Find:
   a. the mean [2]
   b. the median [2]
   c. the mode [1]

4. Given a series of numbers : C, 5, 6, 8, 12, 25, and the mean of these numbers is 15:
   a. Find C [2]
   b. Find the median [2]

5. Given the following collection of integers, where X is unknown:
   3, 2, 7, 2, 2, 4, 5, 2, 4, 6, X
   a. What can be said about the median? [3]
   b. What can be said about the mode? [2]
   c. What can be said about the mean? [2]
   d. If the mean was given as 4, what would be the value of X? [3]

6. Ten students have taken an examination and been given their marks. Student X will not say what his mark is, but the other students have marks of 2, 4, 6, 6, 7, 7, 7, 8, 10. The teacher has told everyone that the mean mark was 6.
   a. What mark did student X obtain? [2]
   b. What mark is the median? [1]
   c. What mark is the mode? [1]

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7. The heights of a group of students from a secondary school are distributed as follows:

Height(m)	1.4<=ht<=1.5	1.5<=ht<=1.6	1.6<=ht<=1.7	1.7<=ht<=1.8	1.8<=ht<=1.9	1.9<=ht<=2.0
Frequency	5	30	36	40	8	1

a. Draw a histogram of this data on graph paper. [4]
b. Calculate the mean and standard deviation. [6]
c. Produce a cumulative frequency table. [2]
d. Draw the cumulative frequency curve. [3]
e. From the cumulative frequency curve, find the median and interquartile ranges. [3]
f. From the graph, determine the probability that the height of a student selected at random will be at least 1.7m. [2]

8. In a class the heights of a graph of students to the nearest centimeter are as follows:

160 - 164 cm → 12 students
165 - 169 cm → 8 students
170 - 174 cm → 10 students
175 - 179 cm → 7 students

a. Plot a table for the above data in order to calculate: [8]
b. the mean height rounded to one decimal places [4]
c. the variance rounded to two decimal places [4]
d. the standard deviation rounded to two decimal places [4]

9. The table shows the results of a survey of 100 students about the number of hours they spent daily in studying for an examination.

No. of study hours	1	2	3	4	5	6
No. of students	15	42	18	10	9	6

a. Draw the histogram for the above data [4]
b. Write down the mode [2]
c. Construct a cumulative frequency table [2]
d. Draw the cumulative frequency curve [3]
e. Estimate to 1 decimal place the median from the curve [2]
f. Find the mean and standard deviation to 2 decimal places [7]

10. The scores of 100 students for their computer studies are as follows:-

Score	1 - 20	21 - 40	41 - 60	61 - 80	81 - 100
No. of students	17	10	27	33	13

a. Plot the above distribution using a Histogram [6]
b. From the Histogram, estimate the Mode [2]
c. Construct a table for the calculation of the Mean and Standard Deviation to 2 decimal places. [7]
d. Construct the Cumulative Frequency table and draw the Cumulative Frequency Curve. [5]

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11. The candidates scores of a random sample of marked examination scripts are shown below : (Individual scores)

26	30	45	53	96	38	87	76
89	18	75	9	35	68	56	28
62	71	54	49	33	77	20	54
6	41	39	76	79	85	93	65
44	65	68	84	84	89	94	18

a. Using class intervals of 1 -10, 11 -20 and so on, produce a frequency distribution table of the above data. [5]

b. On the Graph paper, construct the histogram of the frequency distribution [6]

c. Calculate, by tabulation,

i. the mean score

ii. the standard deviation [9]

12. The distribution of candidates' marks obtained in a test is given by :

Marks	1 - 10	11 - 20	21 - 30	31 - 40	41 - 50	51 - 60	61 - 70	71 - 80	81 - 90	91 - 100
Freq	6	5	8	13	12	18	16	15	11	8

a. On Graph paper draw the Histogram and use it to estimate the MODE. [6]

b. Calculate, by tabulation the

i. mean

ii. standard deviation [8]

c. Construct a cumulative frequency table. [2]

d. Plot the cumulative frequency curve on a graph paper. [4]

13. The distribution of a product's rating obtained in a survey is given by:

Marks	1 - 20	21 - 40	41 - 60	61 - 80	81 - 100	101 - 120	121 - 140
Freq	6	8	14	10	20	22	10

a. On graph paper draw a histogram representing this distribution. [4]

b. Calculate, by tabulation [8]

i. the mean rating to 2 decimal places

ii. the standard deviation to 2 decimal places

c. Construct a cumulative frequency table. [2]

d. Plot the cumulative frequency curve on graph paper. [4]

e. Estimate from the graph:

i. the median [1]

ii. the upper quartile [1]

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14. Forty students run a marathon. The frequency distribution of their times in minutes are as follows:

Time (Minutes)	f
170 - 189	2
190 - 209	8
210 - 229	4
230 - 249	14
250 - 269	3
270 - 289	2
290 - 309	3
310 - 329	1
330 - 349	1
350 - 369	2
	Σf

a. On graph paper, draw a histogram representing the distribution. [4]

b. Construct a cumulative frequency table from your distribution table. [2]

c. Plot the cumulative frequency curve on graph paper. [4]

d. Calculate to 2 decimal places, by tabulating the data in your distribution table:

i. the mean time taken by the students to run the marathon. [4]

ii. the standard deviation [4]

e. The race starter realizes that he made a mistake, and that the times reported are all 5 minutes lower than they should be. How does this affect:

i. the mean? [1]

ii. the standard deviation? [1]

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INFORMATICS

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Chapter 11

Data Structures (I)

Chapter Objectives

At the completion of this chapter, you would have learnt:

- the basic concepts of different data structures;
- how to acquire the ability to relate the types of data structures to real life application.

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11.1 Introduction

A data structure specifies interrelationships among data. Each data structure has unique characteristics that make it different from any other.

The study of data structures involves learning about the various interrelationships that data can have within each unique structure.

It is important for you to understand that the data maybe unchanged, while data structures do change.

The study of data structures is important for system analyst and programmer in designing Database Management Systems, for instance.

11.2 The Concept of Unit Matrix

Constructing of People Matrices for Four Grade of Staff

Below are tables showing the various number of staff for different grade and category for two towns together with their salaries.

By organizing the numbers logically the following matrices are then constructed.

Andover Town.

	Andover					
	Category					
Grade	A	B	C	D	E	F
1	0	2	5	16	0	0
2	2	12	12	27	5	1
3	0	0	24	15	3	2
4	0	0	10	9	2	1

$$\underset{\sim}{A} = \begin{bmatrix} 0 & 2 & 5 & 16 & 0 & 0 \\ 2 & 12 & 12 & 27 & 5 & 1 \\ 0 & 0 & 24 & 15 & 3 & 2 \\ 0 & 0 & 10 & 9 & 2 & 1 \end{bmatrix}$$

Bristol Town.

	Bristol					
	Category					
Grade	A	B	C	D	E	F
1	0	3	18	0	0	1
2	1	4	12	18	2	1
3	0	0	16	14	5	1
4	0	0	8	6	1	0

$$\underset{\sim}{B} = \begin{bmatrix} 0 & 3 & 18 & 0 & 0 & 0 \\ 1 & 4 & 12 & 18 & 2 & 1 \\ 0 & 0 & 16 & 14 & 5 & 1 \\ 0 & 0 & 8 & 6 & 1 & 0 \end{bmatrix}$$

The wage matrix..

	Bristol					
	Category					
Grade	A	B	C	D	E	F
1	1	1	1	1	2	2
2	2	2	3	3	4	5
3	3	3	4	4	6	8
4	8	8	9	9	10	12

$$\underset{\sim}{W} = \begin{bmatrix} 1 & 1 & 1 & 1 & 2 & 2 \\ 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 3 & 4 & 4 & 6 & 8 \\ 8 & 8 & 9 & 9 & 10 & 12 \end{bmatrix}$$

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Addition of Matrices

$$\tilde{A} + \tilde{B} = \tilde{T}$$

The resultant matrix T can be gotten by simply adding the corresponding elements of the individual matrices. Hence by adding the matrices of the two towns the total staffs can be obtained.

$$\underset{\text{Andover}}{\begin{bmatrix} 0 & 2 & 5 & 16 & 0 & 0 \\ 2 & 12 & 15 & 27 & 5 & 1 \\ 0 & 0 & 24 & 15 & 3 & 2 \\ 0 & 0 & 10 & 9 & 2 & 1 \end{bmatrix}} + \underset{\text{Bristol}}{\begin{bmatrix} 0 & 3 & 18 & 0 & 0 & 1 \\ 1 & 4 & 1218 & 2 & 0 & \\ 0 & 0 & 1614 & 5 & 1 & \\ 0 & 0 & 8 & 6 & 1 & 0 \end{bmatrix}} = \underset{\text{Total}}{\begin{bmatrix} 0 & 5 & 23 & 16 & 0 & 1 \\ 3 & 16 & 27 & 45 & 7 & 1 \\ 0 & 0 & 40 & 29 & 8 & 3 \\ 0 & 0 & 18 & 15 & 3 & 1 \end{bmatrix}}$$

11.2.1 Types of Multiplication with Unit Vectors

- ❒ Pre-Multiplication with Row Vector.
- ❒ Post-Multiplication with a Column Vector.

Multiplication of a Matrix with a Row Vector

(Pre-Multiplication of a Matrix with a unit vector)

$$\tilde{U} * \tilde{A} = \tilde{R}$$

To sum up all the people "Vertically" under the respective category (i.e. from A to F) multiply by a row vector.

$$\underset{\text{Row Vector}}{\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}} * \underset{\text{Andover}}{\begin{bmatrix} 0 & 2 & 5 & 16 & 0 & 0 \\ 2 & 12 & 15 & 27 & 5 & 1 \\ 0 & 0 & 24 & 15 & 3 & 2 \\ 0 & 0 & 10 & 9 & 2 & 1 \end{bmatrix}}$$

$$= \begin{matrix} A & B & C & D & E & F \end{matrix}$$
$$= \begin{pmatrix} 2 & 14 & 54 & 67 & 10 & 4 \end{pmatrix}$$

Question: Can the Andover matrix be Pre-Multiply by a column vector?

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Multiplication with Column Vector

(Post-multiplication of matrix with a unit vector)

$$\tilde{R} * \tilde{U} = \tilde{S}$$

To sum up all elements "horizontally" across, the matrix is Post-Multiply by a column vector (i.e. to add 2+14+54+67+10+4).

$$(2\ 14\ 54\ 67\ 10\ 4) * \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = (151)$$

column vector

Question: Can you Post-Multiply by a row-vector?

Calculating the Total Wage Bill

The total wage bill for grade 2 and 3 in category C to E, is obtained by Pre-Multiply by the appropriate row vector.

(Note the one's and zero's in the row and column vector)

CATEGORY

$$[0\ 1\ 1\ 0] * \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \begin{bmatrix} 0 & 5 & 23 & 16 & 0 & 2 \\ 6 & 32 & 81 & 135 & 28 & 5 \\ 0 & 0 & 160 & 116 & 48 & 24 \\ 0 & 32 & 162 & 135 & 30 & 12 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Question: Can you understand the meaning of the multiplication above?

Increasing/Decreasing by Matrix Method

To increase the wages by 5%, 6%, 4%, 8% for the respective grade, set up the matrix 'I' below.

$$\begin{bmatrix} 1.05 & 0 & 0 & 0 \\ 0 & 1.06 & 0 & 0 \\ 0 & 0 & 1.04 & 0 \\ 0 & 0 & 0 & 1.08 \end{bmatrix} * \begin{bmatrix} 0 & 5 & 23 & 16 & 0 & 2 \\ 6 & 32 & 81 & 135 & 28 & 5 \\ 0 & 0 & 160 & 116 & 48 & 24 \\ 0 & 32 & 162 & 135 & 30 & 12 \end{bmatrix}$$

matrix I WAGE MATRIX

Question: Can you understand the logic of matrix I?

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NCC QM DEC. 92 Part Question

Question 8

a. A family buys Milk, Bread and Eggs at unit price of 42,74 and 12 cents respectively the following quantities:

	Milk	Bread	Eggs
Mondays	2	0	6
Tuesday	1	1	12
Wednesday	0	2	0
Thursday	3	3	12

a. Show the quantities as a 4x3 matrix, N, and the prices as a 3x1 matrix P.

b. Calculate each of the following and state what the result means:

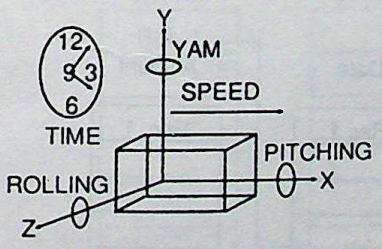
i. A = N*P (3) (3)
ii. B = (1111)*A (2) (2)
iii. C = (1111)*N (2) (2)
iv. D = C*P (2) (2)

11.3 Arrays and Spreadsheets

It is especially easy to manipulate arrays using spreadsheets allow a very quick calculation and re-calculation of large matrices. This allows WHAT IF' analysis to be performed. Try this out during your Lotus 1-2-3 lessons.

Arrays on paper is limited to 2 dimensional but multi-dimensional array can specify on spreadsheets and even in programming languages more than 2 dimensional arrays are required to store related variable. For example in the analysis of aircraft movement such as pitching yaw and rolling.

Aircraft Analysis



How many Dimensional Array Required?
Ans: (1, 2, 3, 4 or 5)

Arrays in Programming Languages

DIM A(25)

NUM[1,2]

Which is Which?
(BASIC, PASCAL)

Another use of the array is the storing numbers and string arrays respectively. String arrays can be operated on by concatenation of the elements in the respective arrays.

Array addresses, in the computer can be assigned by having two bytes for each number. For an array size of 4*7, the total number of bytes 4*7*2 = 52, the first quarter 14 bytes are the first row and next 14 bytes for the next row and so on.

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11.3.1 Elementary Array Operations

(Examples of the use of arrays in Pascal programming)

Student Number	Science	Geography	History	Mathematics
1307	46	58	24	49
1314	71	42	54	83
1318	24	17	20	19
1321	83	46	53	78
1327	75	58	62	88
1334	40	32	6	47

Example : {This program fills the array row by row}

```
For Row : = 1 to 6 Do
        For Col : = 1 to 5 Do
            Readln(Table [Row, Col]);
```

Example : {This program fills the array column by column}

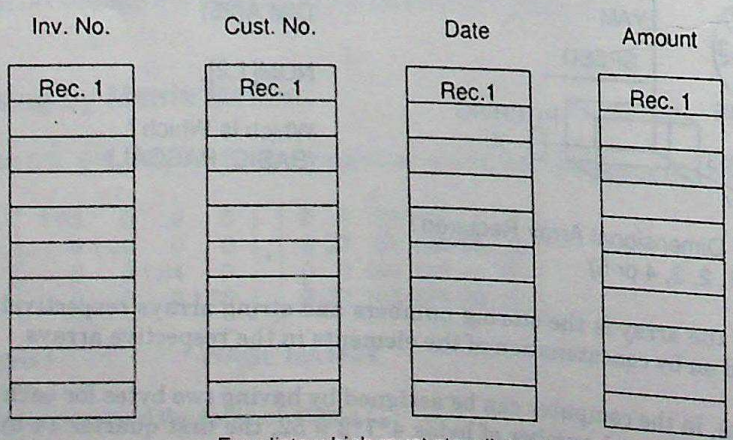
```
For Col := 1 to 5 Do
        For Row : 1 to 6 Do
            Readln(Table [Row,Col]);
```

11.4 Lists

A record consists of different data items stored in an array.

Below is a list consisting of four data items that are not linked. An example is given below.

Four lists which must stay 'linked'



Four lists which must stay 'linked'

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An Example of a Linked Record

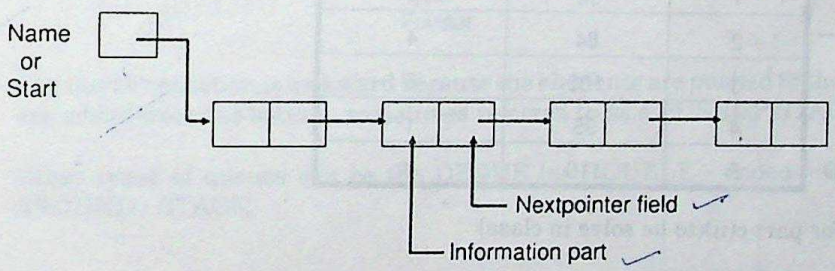
Try the following quiz by matching the device to the explanation. This will provide a better understanding of linked records.

1.	Modem	A.	A device used to receive signals from one circuit and automatically repeat and amplify and reshape the signals to another circuit.
2.	Front-End Processor	B.	A system which removes some of the processing load from a central computer, typically handling communication coordination functions before the data is sent to the central system for processing.
3.	Repeater	C.	Devices needed at the ends of an analogue transmission line to translate between the digital code of computers and the analogue wave form; an abbreviation for modulation - demodulation.

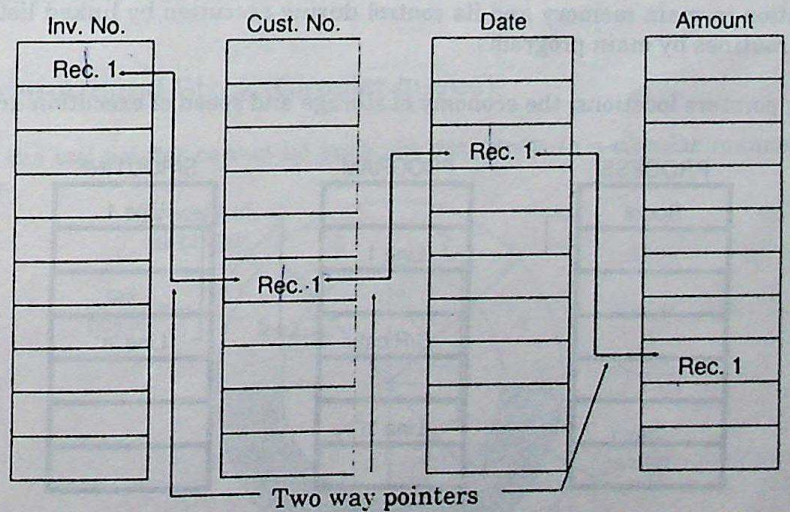
11.5 Linked List

A linked list, or one-way list, is a linear collection of data elements, called nodes, where the linear order is given by means of pointers. That is. each node is divided into two parts : the first part contains the information of the element, and the second part, called the link field or next pointer field, contains the address of the next node in the list.

Example : Linked list with 3 nodes



A more useful application of a list is a linked list. The Invoice Number, Customer Number, Date and Amount are linked by a two way pointer.



The lists after ordering

Accessing any of the data items allow the access to the others

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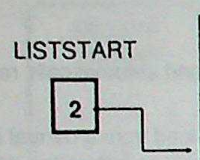
Example

A LINKED TABLE holding KEY numerical values in ASCENDING order of which the POINTER 'LISTSTART' holds the value 2, contains:

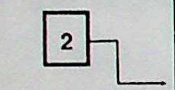


INDEX	KEY	LINK TO NEXT RECORD
1	96	3
2	84	1
3	105	-1

i. Re-write the table as it would appear if we added, at INDEX 4 THE KEY 95 and KEY 110 at INDEX 5 (4)

ii. Show the original table as it would appear if KEY 96 were DELETED (2)

- solution for part c(i);



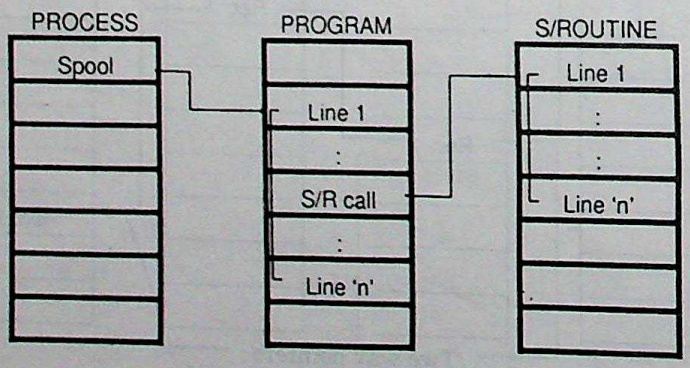
INDEX	KEY	LINK TO NEXT RECORD
1	96	3
2	84	4
3	105	5
4	95	1
5	110	-1

- solution for part c(ii)(to be solve in class)

11.5.1 System Lists

Program location in main memory and its control during execution by linked list for example calling of subroutines by main programs.

By managing pointers locations, the economy of storage and speed of execution are achieved.



Program Lists with pointers

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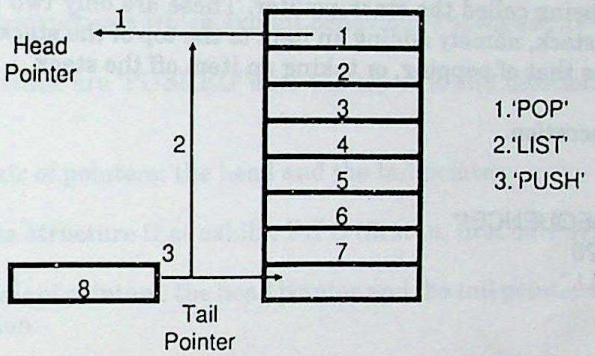
11.6 Queues

The queue is yet another data structure which differs in a number of ways from the stack. It may be used in real-time processing, for scheduling of jobs in the context of an operating systems or in other similar applications such as data transfer.

It is a FIFO (First-In-First-Out) structure which means that the first item to enter the queue is the first item to leave and new items always get added to the end of the queue.

As with the stack we can represent a queue by a one-dimensional array with the need for two pointers. The first to indicate the front of the queue and the other indicate the next space capable of holding an element joining the queue, these being the head and tail pointers respectively.

A method of inserting and retrieving data similar to a stack but on 'FIRST-IN-FIST-OUT'.



The queue operation is awkward because the elements are pushed to the front and new elements are added from the bottom, sometimes referred to as a PUSHUP STACK or LIST.

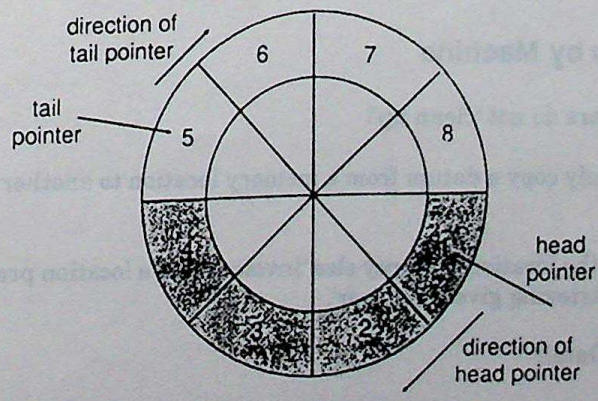
Other types of queues can be the DEQUE (or DOUBLE - ended - Queue) and the WRAP - AROUND - STACK.

11.6.1 DEQUE

Data are push and pop (add and delete) from both ends.

11.6.2 Wrap-Around-Stack (Circular queue)

In this type the tail pointer cannot be push out but moves in a circular manner ahead of the head pointer.



Wrap-Around Stack

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11.7 Stacks

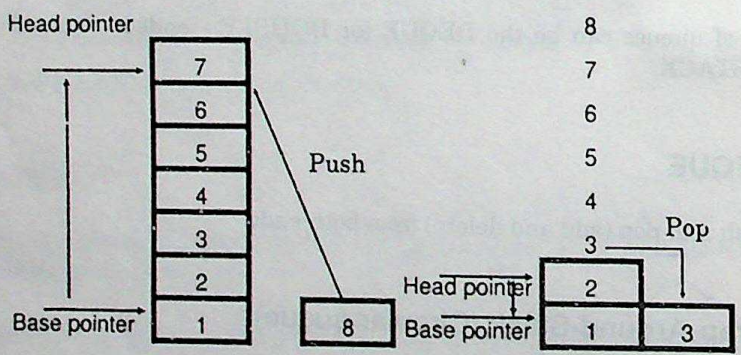
The stack is an example of a data structure found quite commonly in computing which is especially used in system software. They may by used to carry out stage of a calculation, for handling interrupts in translation programs or to pass information from a main program to a subprogram.

The main physical characteristics of a stack is that it is a LIFO (Last-In-First-Out) structure. It can be liken to a pile of plates in which it is easy (if sometime a little unstable) to add extra plates to the top. Plates can also be easily removed from the top of the pile, but not by extrication from the bottom of the pile.

We can represent a stock as a one-dimensional array with two pointers, one pointing to the base of the stack (to identify the first element) and the other pointing to the position of the first available element, this being called the stack pointer. These are only two possible operations that can be done with a stack, namely adding an item to the top of the stack (called stacking or pushing) and the other is that of popping, or taking an item off the stack.

An example of Stack Operation.

```
DIM S(50)
REM **PUSHING SEQUENCE**
IF P <= 50 THEN 120
PRINT "STACK FULL"
STOP
LET S(P) = E
LET P = P+1
GOTO 90
:
```



Push and Pop operations

11.7.1 Corruption by Machine

Machines and computers do not 'clean up'!

A 'POP' operation merely copy a datum from a memory location to another leaving the previous datum intact.

For a push make sure the location is empty else 'Invoice'; into a location previously held a longer string 'calendar' on retrieving gives 'Invoicer'.

Hence Corruption of Data!!

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Points to Remember

- An array is a storage area in primary memory. Each data entry stored in an array is accessed with a single subscript or index value.
- Arrays can be one-dimensional or multi-dimensional.
- Arithmetic operations applied to a two-dimensional table result in a matrix.
- Linked list are data structures that permit records in a file or entries in an array to be maintained in logical order without remaining in physical order.
- Linked list use a pair of pointers: the head (list) pointer for the start of the list and tail pointer for the last entry.
- Stacks are data structures which exhibit behaviour known as LIFO (last in, first out).
- Additions to a stack are 'PUSHED' onto the top of it and deletion are 'POPED' singly off the top.
- Stacks use a pair of pointers: the head and the tail pointer.
- Queues are data structure that exhibit FIFO (first in, first out) behaviour.
- Queues use a pair of pointers: the head pointer and the tail pointer which indicates the next available location.

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Past Years Questions

Please refer to Chapter 12

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Chapter 12

Data Structures (II)

Chapter Objectives

At the completion of this chapter, you would have learnt:

- how to acquire the ability to manipulate or to use the different types if data structures;
- how to construct and traverse a binary tree;
- comparison of data structures.

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12.1 Introduction

So far, we have been studying mainly linear types of data structures: strings, arrays, lists, stacks and queues. This sections defines a nonlinear data structure called a tree. This structure is mainly used to represent data containing a hierarchical relationship between elements. E.g. records, family trees and tables of contents.

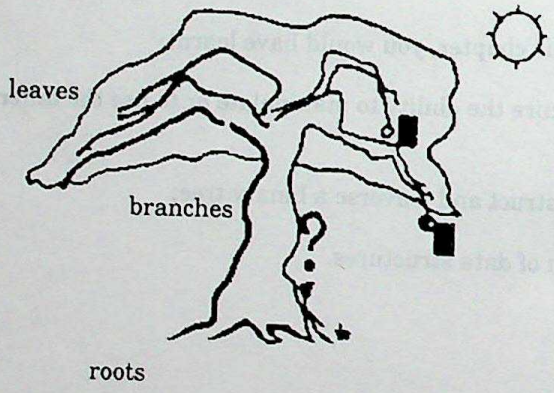
First we investigate a special kind of tree, called a binary tree, which can be easily maintained in the computer. Although such a tree may seen to be very restrictive, we will see later in the section that more general trees may be viewed as binary trees.

12.2 Tree

Natural Tree

What has a natural tree got to do with a computer?

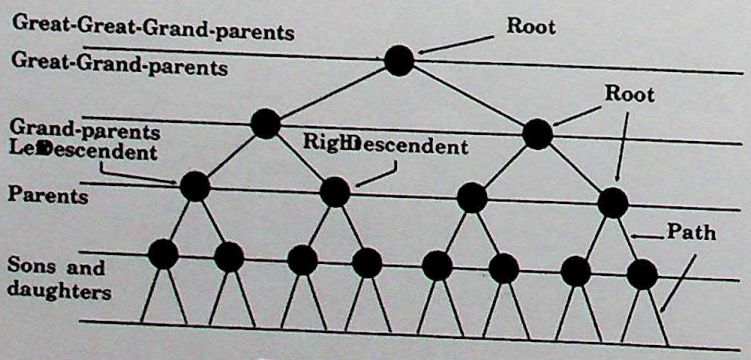
Try a pseudo-code to get the apples and return to the ground.



A natural tree

Computer tree

A simplified model of a real tree. Where each node has at most two others below it. This is a binary tree.



The family tree model

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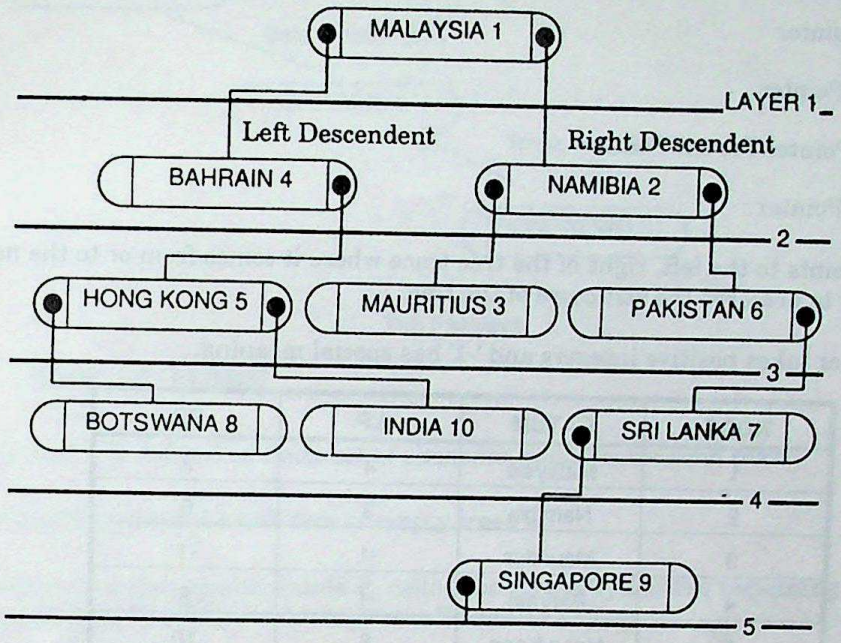
The countries Malaysia, Namibia, Mauritius, Bahrain, Hong Kong, Pakistan, Sri Lanka, Botswana, Singapore and India are to be placed in an alphabetical order using pointers.

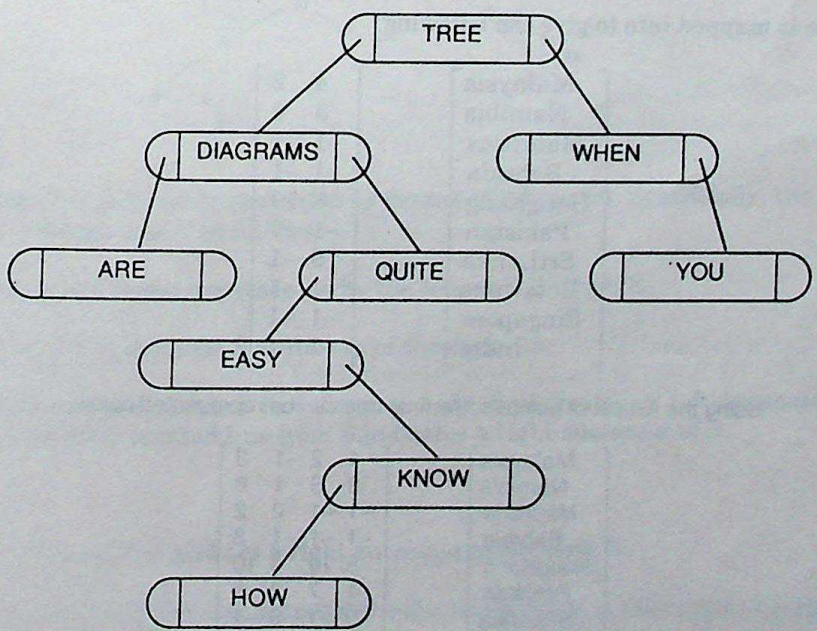
The diagram below is a complete, 'linkages' of all the countries.



The tree diagram completed

Further Example of Tree Diagram

The text "TREE DIAGRAMS ARE QUITE EASY WHEN YOU KNOW HOW." is to be represented in the form of a tree.



Question: Can you understand the logic of the binary tree?

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12.3 Tables

In tables, there are basically 4 types of pointers

i. Left Pointer

ii. Right Pointer

iii. Back Pointer/Parent Pointer

iv. Trace Pointer

The Pointers, points to the left, right of the tree trace where it comes from or to the next node of the tree. This to preserve the structure of the tree.

The node number takes positive integers and '-1' has special meaning.

NODE	DATUM	LP	RP
1	Malaysia	4	2
2	Namibia	3	6
3	Mauritius	-1	-1
4	Bahrain	-1	5
5	Hong Kong	8	10
6	Pakistan	-1	7
7	Sri Lanka	9	-1
8	Botswana	-1	-1
9	Singapore	-1	-1
10	India	-1	-1

The above tree is mapped into to give the following;

$$\begin{bmatrix} \text{Malaysia} \\ \text{Namibia} \\ \text{Mauritius} \\ \text{Bahrain} \\ \text{HongKong} \\ \text{Pakistan} \\ \text{SriLanka} \\ \text{Botswana} \\ \text{Singapore} \\ \text{India} \end{bmatrix} \quad \begin{bmatrix} 4 & 2 \\ 3 & 6 \\ -1 & -1 \\ -1 & -1 \\ 8 & 10 \\ -1 & 7 \\ 9 & -1 \\ -1 & -1 \\ -1 & -1 \\ -1 & -1 \end{bmatrix}$$

Using the full set of pointers, the final tree can be represented below,

$$\begin{bmatrix} \text{Malaysia} \\ \text{Namibia} \\ \text{Mauritius} \\ \text{Bahrain} \\ \text{HongKong} \\ \text{Pakistan} \\ \text{SriLanka} \\ \text{Botswana} \\ \text{Singapore} \\ \text{India} \end{bmatrix} \quad \begin{bmatrix} 4 & 2 & -1 & 3 \\ 3 & 6 & 1 & 6 \\ -1 & -1 & 2 & 2 \\ -1 & -1 & 1 & 8 \\ 8 & 10 & 4 & 10 \\ -1 & 7 & 2 & 9 \\ 9 & -1 & 6 & -1 \\ -1 & -1 & 5 & 5 \\ -1 & -1 & 7 & 7 \\ -1 & -1 & 5 & 1 \end{bmatrix}$$

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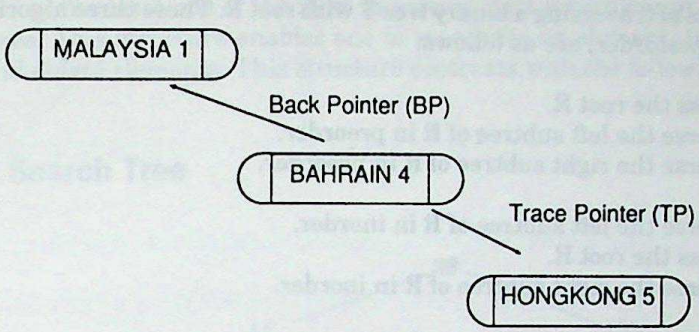
The Back / Trace Pointer



The Pointers

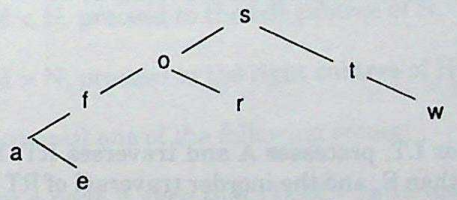
12.4 Binary Trees

A binary tree T is defined as finite set of elements, called notes such that:

- T is empty (called the null tree or empty tree), or
- T contains a distinguished node R, called the root of T, and the remaining nodes of T form an ordered pair of disjoint binary trees T_1 and T_2.

If T does contain a root R, then the two trees T_1 and T_2 are called, respectively, the left and right subtrees of R. If T_1 is non-empty, then its root is called the left successor of R; similarly, if T_2 is non-empty, then its root is called the right successor of R.

Example: Software



A binary tree T is frequently presented by means of a diagram. Specifically, the diagram above represents a binary tree T as follows.

- T consists of 8 nodes, represented by the letters S through E.
- The root of T is the node S at the top of the diagram.
- A left-downward slanted line from a node S indicates a left successor of S and a right-downward slanted line from S indicates a right successor of S.

Observe that:

- O is a left successor and t is a right successor of the node S.
- The left subtree of the root S consists of the nodes o, f, r, a and e, and the right subtree of s consists of the nodes t and w.

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12.5 Traversing of Binary Trees

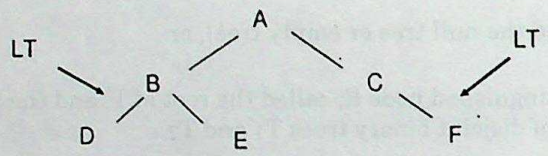
There are three standard ways of traversing a binary tree T with root R. These three algorithms, called preorder, inorder and postorder, are as follows:

Preorder:
1. Process the root R.
2. Traverse the left subtree of R in preorder.
3. Traverse the right subtree of R in preorder.

Inorder:
1. Traverse the left subtree of R in inorder.
2. Process the root R.
3. Traverse the right subtree of R in inorder.

Postorder
1. Traverse the left subtree of R in postorder.
2. Traverse the right subtree of R in postorder.
3. Process the root R.

A simple example of Binary Tree



12.5.1 Preorder Traversal

The preorder traversal of T processes A, traverses LT and traverses RT. However, the preorder traversal of LT processes the root B and then D and E, and the preorder traversal of RT processes the root C and then F. Hence ABDECF is the preorder traversal of T.

12.5.2 Inorder Traversal

The inorder traversal of T traverses LT, processes A and traverses RT. However, the inorder traversal of LT processes D, B and then E, and the inorder traversal of RT processes C and then F. Hence DBEACF is the inorder traversal of T.

12.5.3 Postorder Traversal

The postorder traversal of T traverses LT, traverses RT, and processes A. However, the postorder traversal of LT processes D, E and then B, and the postorder traversal of RT processes F and then C. Accordingly, DEBFCA is the postorder traversal of T.

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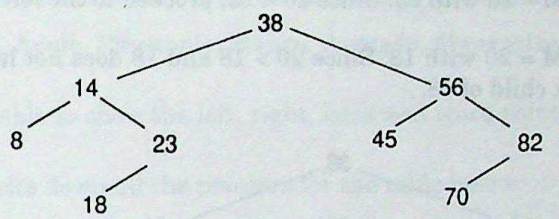
12.6 Binary Search Trees

This section discusses one of the most important data structures in computer science, a binary search tree. This structure enables one to search for an element. It also enables one to easily insert and delete elements. This structure contrasts with the following structures:

Binary Search Tree



12.7 Searching and Inserting in Binary Search Trees

Suppose T is a binary search tree. This section discusses the basic operations of searching and inserting with respect to T. In fact, the searching and inserting will be given by a single search and insertion algorithm.

Suppose an ITEM of information is given. The following algorithm finds the location of ITEM in the binary search tree T, or inserts ITEM as a new node in its appropriate place in the tree.

a. Compare ITEM with the root node N of the tree.

   i. If ITEM < N, proceed to the left subtree of N.

   ii. If ITEM > N, proceed to the right subtree of N.

b. Repeat Step (a) until one of the following occurs:

   i. We meet a node N such that ITEM = N. In this case the search is successful.

   ii. We meet an empty subtree, which indicates that the search is unsuccessful, and we insert ITEM in place of the empty subtree.

In other, proceed from the root R down through the tree T until finding ITEM in T or inserting ITEM as a terminal node in T.

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Example:

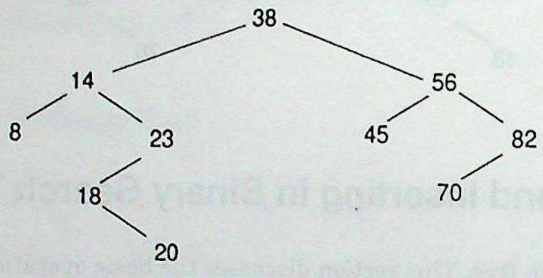
Consider the binary search tree T above. Suppose ITEM = 20 is given. Simulating the above algorithm, we obtain the following steps:

1. Compare ITEM = 20 with the root, 38, of the tree T. Since 20 < 38, proceed to the left child of 38, which is 14.

2. Compare ITEM = 20 with 14. Since 20 > 14, proceed to the right child of 14, which is 23.

3. Compare ITEM = 20 with 23. Since 20 < 23, proceed to the left child of 23, which is 18.

4. Compare ITEM = 20 with 18. Since 20 > 18 and 18 does not have a right child, insert 20 as the right child of 18.



Points to Remember

- Binary trees are non-linear data structures in which the root and each node on the tree contain a maximum of two children.
- A binary tree can be created within an array using left and right child pointers similar to linked list concepts
- Additions, changes and deletions of a left node are fairly easy
- The 4 types of pointers in a binary can be represented in a form of a table
- Three traversal of binary search tree, Preorder, Inorder and Postorder.
- A comparison

	Sorted Linear Array	Linked List	Binary Tree
Search	Quick(binary Search)	Slow(Linear Search)	Quick(Binary Search)
Insert & Delete	Expensive	Easy	Easy

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Past Years Questions

1. Produce a binary tree diagram of the following data:

Node	1	2	3	4	5	6	7	8
Content	C	J	P	T	K	B	M	S

[4]

2. a. Construct a dictionary-order binary tree for the fruits below:

Durian, Apple, Pineapple, Mango, Avocado, Mangosteen, Watermelon, Pear. [8]

b. Use a table to show the left, right, back and trace pointers for the above tree. [8]

c. i. Write down all the pointers for the table below: [4]

Note: that contents are to be arranged in ascending order.

Address	Contents	Pointer
4	101	6
2	90	4
5	85	2
6	102	7
7	105	-2

ii. Re-arrange the pointers if contents 102 and 85 are removed. [2]

3. Given the following items in series, construct the dictionary-order binary tree. [4]

Malaysia, India, Singapore, Bahrain, China, Turkey,Pakistan, Hong Kong

4. a. i. Construct the binary tree for the following data :

Keyboard, CPU, Mouse, Monitor, Scanner, Disk, Cables, Drives, Printer, Software [6]

ii. From the above binary tree, build a table to show its Left, Right, Back (parent).

Use -1 to represent a null value. [6]

b. i. Write down the pointers for the table below, where elements are linked in numerical order: [2]

Index	Key	Linked to next record
1	50	
2	100	
3	87	
4	34	

ii. Modify the table to reflect the additional of index 5 with key value 66 and index 6 with key value 104. [4]

iii. Modify the table to reflect the deletion of index 2, key 100. [2]

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5. Given the following items:

Hockey, Baseball, Soccer, Tennis, Squash, Golf

a. Construct an alphabetically-ordered binary tree [3]

b. From the binary tree, build a table with Left and Right Pointers [2]

6. A table to represent the contents of a BINARY TREE is:

Node	Contents	Left Pointer	Right Pointer
1	Gooch	2	4
2	Atherton	-1	3
3	Botham	-1	-1
4	Gower	-1	5
5	Smith	-1	-1

a. Draw the binary tree which this information represents. [5]

b. Show the changes to the TABLE when *Fraser* is added to the contents. [2]

c. Explain how a BINARY TREE can be used to SORT a large amount of data amount of data by computer. [3]

7. a. Produce a BINARY TREE diagram using the data below:

Node	1	2	3	4	5	6	7	8	9	10	11
Contents	Rice	Soya	Tea	Coffee	Melon	Yams	Bead	Potatoes	Cocoa	Sugar	Bananas

b. Represent the tree in the form of a TABLE including columns for the LEFT and RIGHT pointers. [3]

c. Show how the TABLE would be used to search for SUGAR. [3]

d. Show the changes to the TABLE when MILK is added to the tree. [3]

e. State two reasons why a BINARY TREE is often preferred to a linear list for holding data. [2]

f. If a tree is ideally shaped with 1,000 nodes, what is the MAXIMUM number of nodes you would expect to check when searching for a given value? Briefly justify your answer. [2]

8. a. Produce a Tree diagram using the data below: [6]

Node	1	2	3	4	5	6	7	8	9	10
Contents	Malaysia	India	Kuwait	Nigeria	Bahrain	Pakistan	Namibia	Singapore	Hong Kong	Australia

b. Make a table from you tree, ADD Left and Right

Pointer columns and complete the table. [3]

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9. A baker sells bread to ten shops every day. When he de livers the bread in the morning, he collects the bread which was not sold the previous day, and gives it to the local zoo for their animals. One day, he gets the following number of loaves returned:

ShopA	3
ShopB	6
ShopC	2
ShopD	8
ShopE	0
ShopF	4
ShopG	7
ShopH	5
ShopI	12
ShopJ	1

a. Construct a binary tree for the above data, such that traversing the tree will list the shops in increasing order of loaves returned. Mark each node in the tree with both the shop name and the number of loaves e.g. A(3) [8]

b. For your binary tree, build a table to show each node's left, right and back pointer. Use -1 to represent a null pointer. [6]

c. Redraw your table to reflect the deletion of shop I from the tree. [3]

d. Redraw your table to reflect the further deletion of shop D from your tree. [3]

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Chapter 13

Recursion

Chapter Objectives

At the completion of this chapter, you would have learnt:

- the definition of recursion;
- the definition of Mutual Recursion;
- the steps to make a Recursive Routine Work;
- the steps to search Binary Trees Recursively;
- the steps to Search a List Recursively;
- to trace a Simple Recursive Function.

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13.1 Definition of Recursion

- A process of a function or procedure referencing itself.
- The product of any step of an iteration is 'fed back in' to the calculation for the succeeding step.

13.2 Definition of Mutual Recursion

- Also known as indirect recursion
- Recursion in which a subprogram references other subprograms and in which some chain of subprogram references eventually results in a reference to the first subprogram again.
- E.g. of a mutual recursion as shown below

```
Function second(a, b:real):real;forward;

Procedure first(x,y:real);
Var w:real;
Begin
    .
    .
    w:=second(x,y);
    .
    .
End;

Function second;
Var u,v:real;
Begin
    .
    .
    first(u,v);
    .
    .
    second:=...
End;
```

- In the above example, a dummy declaration for second is necessary so that first can access second before second has been declared. The true declaration for second then follows the declaration for the first.

Steps to Make a Recursive Routine Work

- When writing a recursive procedure or function, it is essential that the procedure (or function) include some terminating condition. This prevents the recursion from continuing indefinitely. Having an "infinite loop" in a program is a long-standing hazard.
- In order to stop the procedure when it has gone far enough for our purpose, we must first define, with the user, what that purpose is and to what level of accuracy of estimate the answer is to be calculated.

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Steps to Search Binary Trees Recursively

- A binary tree either
  - is empty or
  - consists of a node called the root, which as pointers to two disjoint binary (sub)trees called the left and the right subtree.

The first operation for binary trees that we consider is traversing the tree, that is, visiting each node in the binary tree exactly once. There are three basic steps in traversing a binary tree recursively.

- The tree basic steps are as follows:
  1. visit a node
  2. traverse the left subtree of the node and
  3. traverse the right subtree of the node.

- These three steps can be performed in any order. If we denote these steps by

  N : visit a node

  L : traverse the left subtree

  R : traverse the right subtree

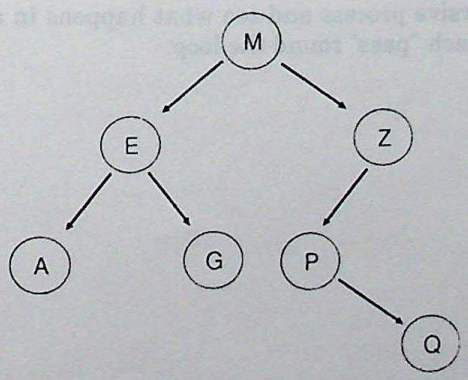
- Then the six different orders are NLR, LNR, LRN, NRL, RNL, RLN. We can reduce this list to the first three orders if we agree always to traverse the left subtree before the right subtree. The traversals that result from these three orders are given special names:

  NLR : preorder traversal

  LNR : inorder traversal

  LRN : postorder traversal

- To illustrate these traversals, consider the following binary tree:



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- The three types of traversals visit the nodes in the following sequences:

  preorder : M E A G Z P Q

  inorder : A E G M P Q Z

  postorder : A G E Q P Z M

Steps to Search a List Recursively

- The general pattern common to recursive algorithms that deal with linked lists are as follows :

  1. Process the current node.
  2. Recursively process the remaining nodes.

- An example of a linked list using recursive function is as follows:-

```
Function Number of nodes(list:nodepointer): Wholenumbers;
Begin
    If list = nil then
        numberofnodes := 0
    else
        numberofnodes := 1 + numberofnodes(list^.next)
```

In this example, the recursive function, number of nodes, is a one-way linked list and the number of nodes can be computed by 1 + numberofnodes. That is, visit a node (by this we mean add 1 to a running count) and then recursively process the remaining nodes in the linked list. The base case becomes what should be added to the running count when we fall off the end of the list (we add 0, obviously).

The simplicity if this algorithm is a characteristic of most recursive algorithm. Notice also how this algorithm follows the general two-step pattern of "process the current node" then "recursively process the remaining nodes" that we mentioned earlier.

Tracing a Simple Recursive Function

Let us examine a simple recursive process and see what happens in a pencil and paper "run" and tabulating the results of each "pass" round the loop.

```
i=1, d=1, si=1

Accept n
    Do while i  <= n
        print i, si
        i ← i+1
        d ← d+2
        si ← si+d
    Enddo
End
```


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Pass	d	Print i	Print s_i
1	1	1	1
2	3	2	4
3	5	3	9
4	7	4	16
5	9	5	25
6	11	6	36

- This table contains some repetition since "I" is repeated in the printout but it is the pattern of the table which we should follow. It will, in many cases, be necessary to do more than six "sample" passes in a run but six is enough for us here. What does the "print" information tell us? What function maps the "i" onto the "s_i"?

- Clearly it is the table of natural squares which we are developing. Let us take only the "print columns and add a differences (Δ) column so that we may fully explore the iteration.

- The table including differences:

i	0	1	2	3	4	5	6
s_i	0	1	4	9	16	25	36
Δ	0	1	3	5	7	9	11

and we see that the sequence of differences is the set of odd numbers. In other words, the natural squares of numbers up to "n" may be found by adding all the odd numbers from 1st to nth. It is not difficult to see how the table would continue.

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